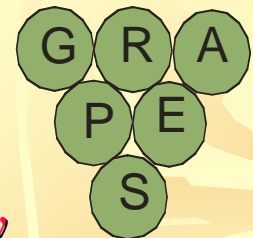


# Toward a mechanistic approach of macroeconomy

**Marcel Ausloos**  
*7 rue des Chartreux, B-4122 Plainevaux*

*Previously at*  
*Group for Research and Applications of Physics in Economy and Sociology*



# Community extraction through network structural analysis

*with*

*Mircea Gligor, **Neamt, Romania***

*Janusz Miskiewicz, **Wroclaw, Poland***

*Francisco Redelico & Araceli Proto, **Buenos Aires, Argentina***

# *Fluctuations and Correlations*

.....

*Countries characterized by macro-economic indicators*

.....

*A cluster expansion-like method*

.....

*(Evolving) Weighted Networks  
with Vector-like Nodes*

.....

*Globalization through distance correlations*

# *Please read/see : (1)*

- *M. Gligor & M. A., Cluster structure of EU-15 countries derived from the correlation matrix analysis of macroeconomic indices fluctuations, Eur. Phys. J. B 57 (2007) 139-146*
- *M. Gligor & M. A., Convergence and cluster structures in EU area according to fluctuations in macroeconomic indices, J. Econ. Integration 23 (2008) 297-330*
- *M. A. & M. Gligor, Cluster Expansion Method for Evolving Weighted Networks Having Vector-like Nodes, Acta Phys. Polon. A 114 (2008) 491-499*
- *M. Gligor & M. A., Clusters in weighted macroeconomic networks : the EU case. Introducing the overlapping index of GDP/capita fluctuation correlations, Eur. Phys. J. B 63 (2008) 533-539*
- *F. O. Redelico, A.N. Proto, & M. A., Hierarchical structures in the Gross Domestic Product per capita fluctuation in Latin American countries, Physica A 388 (2009) 3527-3535*

# *Please read/see : (2)*

*Follow up of studies  
on correlations between GDPs of rich countries*

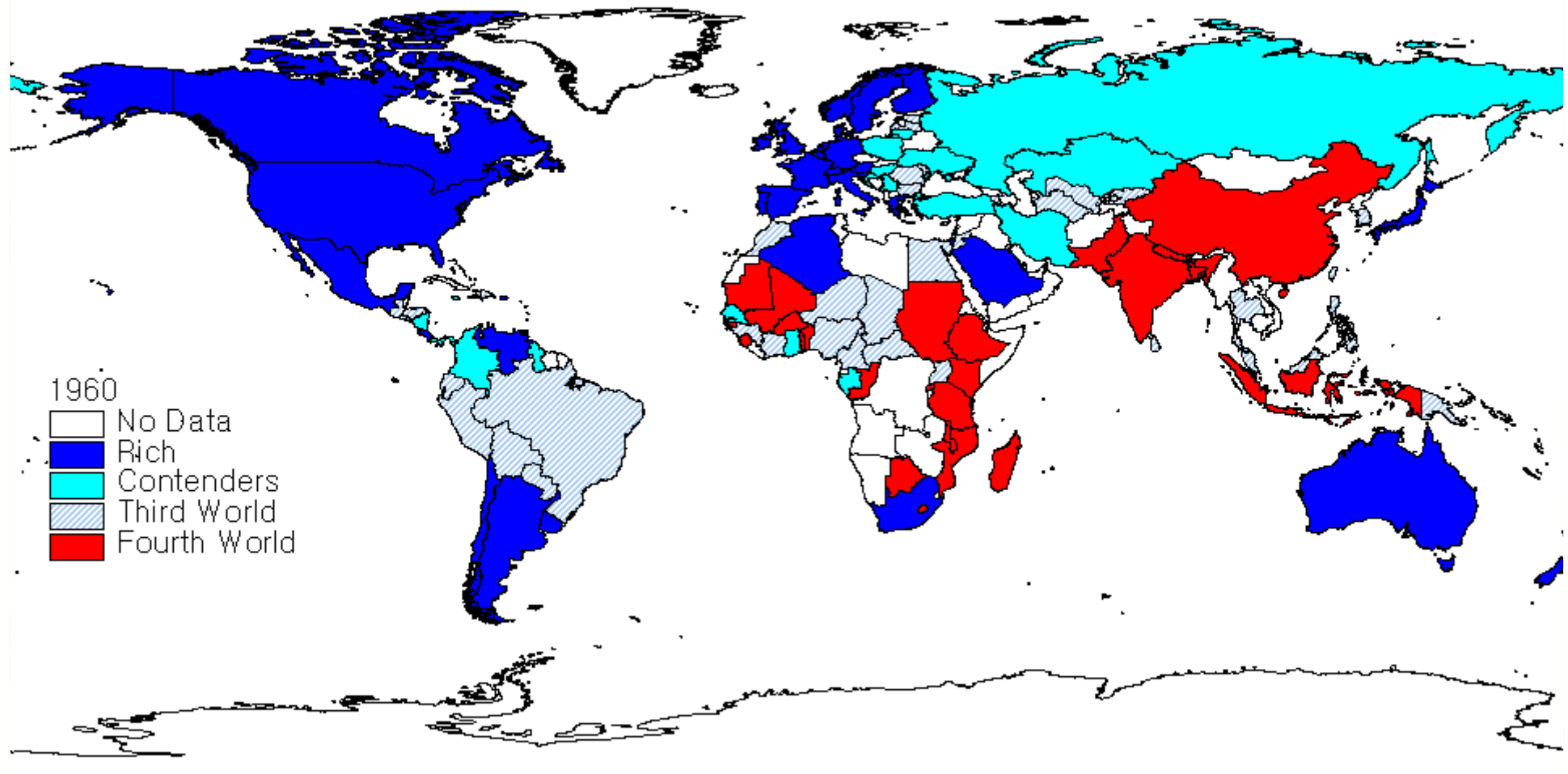
- *J. Miskiewicz & M. A., An attempt to observe economy globalization: the cross correlation distance evolution of the top 19 GDP's, Int. J. Mod. Phys. C 17 (2006) 317-332*
- *M. A. & R. Lambiotte, Clusters or networks of economies? A macroeconomy study through GDP fluctuation correlations, Physica A 382 (2007) 16-21*
- *R. Lambiotte & M. A., N-body decomposition of bipartite networks, Phys. Rev. E 72 (2005) 066117 (8 pages)*

# Macroeconophysics

- Questions ?
- Agents ?
  - In microeconophysics : fundamentalists/chartists
  - In macroeconophysics : ?
- Level of modelization
  - Less data : short time series
    - GDP released quaterly, adjusted monthly,
    - Stationarity ? (... Transitivity)
  - Data not so standardized

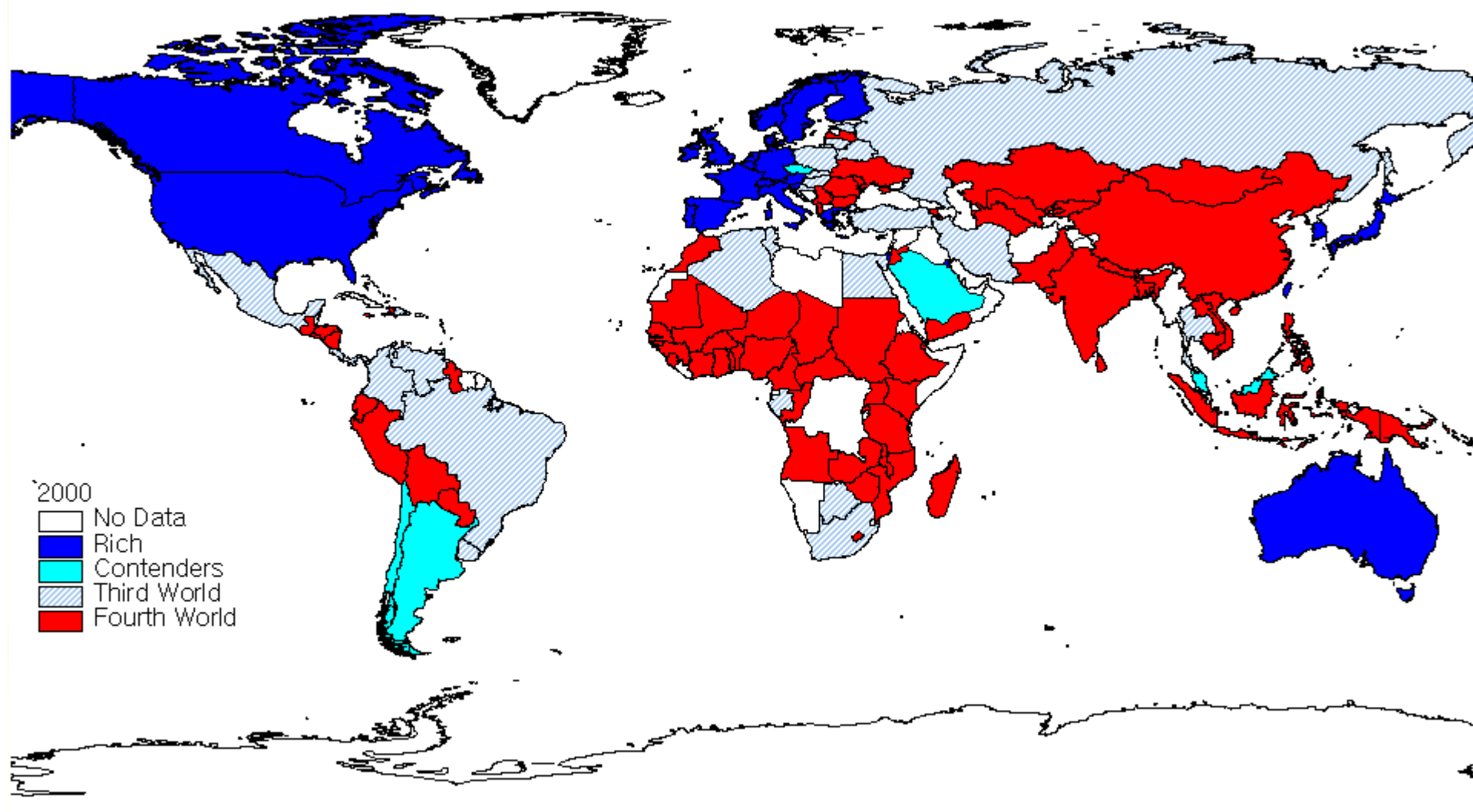
Major aim : « Hamiltonian construction »

The Four Worlds in 1960 ←





The Four Worlds in 2000 ←





## *The border countries and their GDP per capita levels (in \$PPP, 1995 prices)*

	1960	1978	2000
First to second	Portugal (3205) Croatia (3085)	Portugal (7993) Puerto Rico (7662)	Greece (13821) Barbados (13297)
Second to third	Haiti (2139) Malaysia (2120)	Armenia (5294) Fiji (5156)	Malaysia (9887) Slovak (8595)
Third to fourth	Nigeria (1080) Madagascar (1031)	Guyana (2728) Cote d'Ivoire (2649)	Egypt (4630) Bulgaria (4313)

# PPP, EKS, GK,

- *PPP are interspatial indices constructed for the purpose of comparing currencies and volumes across countries,*
  - *just as a consumer price index measures the cost of a basket of goods and services over time.*
- *Elteto-Koves-Szulc (EKS) method*
- *Geary Khamis (GK) method*
- *The EKS PPP result from the geometric average of the direct PPP between a pair of countries and all the indirect PPP derived through third countries,*
  - *with the direct PPP having twice the weight of each indirect PPP*

# Content (1) : globalization question

- toward a Globalization “conclusion” :
  - distance correlations  $\Rightarrow$  network diameter/size
  - « globalization limit » ? ... !
- *Globalization process in economy is understood as an increase of similarities within (macroeconomy) development patterns*
- Various correlation measures
- Various distance definitions
- Theil index
  - q-Theil index

# Content (2) : GDP correlations

- How to extract clusters and/or communities through a network analysis
  - Have a mechanistic interpretation
- GDP/capita (GDP/c) correlations investigated in various time windows (T),
  - for the time interval 1990-2005.
- Target group of countries : three sets based on
  - 25 European (EU), 18 Latin American (LA) countries, USA
  - EU25: LA18: EU25&USA: LA18&USA; EU25&LA18; EU25&LA18&USA

# Content (3) : Network construction

- Links are weighted edges based on the *coherent* Pearson correlation coefficient (CPCC) between the countries GDP/c
  - measured over a specific T
  - N.B. Possible time lag effects are to be examined
- Define an *average overlap index* (AOI) for each node.

# Content (4) : network analysis

- The characteristics of each fully connected *weighted* network evolve with time !
- How to extract clusters and/or communities through a network analysis ?
- To illustrate cluster appearance and stability a double attrition process is applied:
  - on the nodes, through the AOI ranking,
  - on the links, through the CPCC values.
- ... the *vector node network*: a generalisation taking into account various macroeconomic indices
  - Tagged nodes => Hamiltonian-like with “interactions”

# Content (5): PCA

- PCA: Principal Component Analysis
- Statistically relevant eigenvalues and eigenvectors of the CPCC adjacency matrix



# 1. Data

Macro-economy Indicators :

- Gross Domestic Product (GDP)
- Final Consumption Expenditure (FCE)
- Gross Capital Formation (GCF)
- Net Exports (NEX)
- ...
  - GDP/capita
  - GDP/worked hour
  - Consumer Price Index
  - Interest Rates of the Central Banks
  - Labour Force
  - Unemployment
  - Gini coefficient

# MEI

## ***Gross Domestic Product (GDP)***

- = sum of gross value added by all resident producers plus product taxes and minus subsidies

## ■ ***Final Consumption Expenditure (FCE)***

- = sum of household and government final consumption expenditure

## ■ ***Gross Capital Formation (GCF)***

- = outlays on additions to the fixed assets plus net changes in the inventories

## ■ ***Net Exports (NEX)***

- = Exports - Imports

# Data source

<http://www.ggdc.net/databases/hna.htm>

<http://go.worldbank.org>

<http://devdata.worldbank.org/query/default.htm>

*Time interval 1990 - 2005*

▪ EU-25 +

$M = \dots$

▪ LA-18 +

▪ USA

Notations:

Roots Web Surname List

<http://helpdesk.rootsweb.com/codes/>

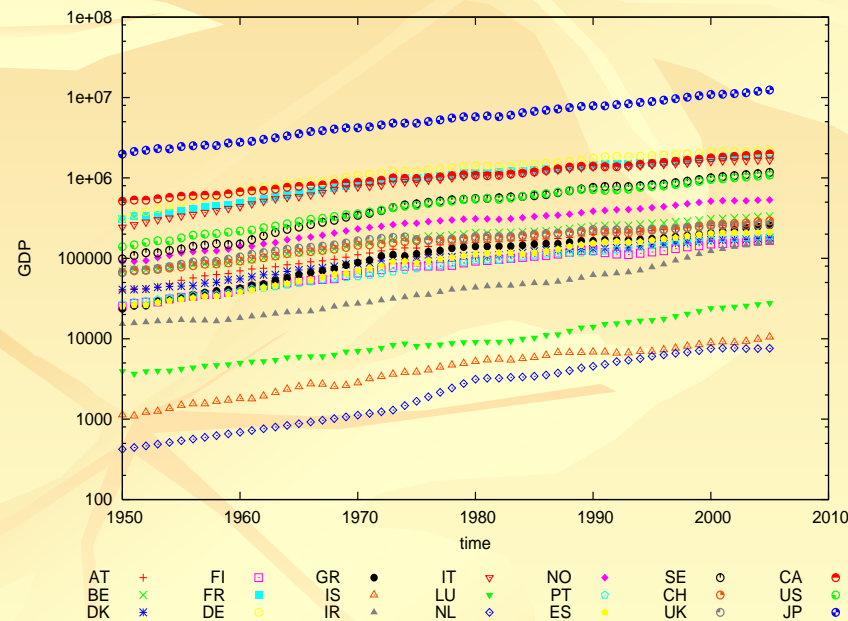
# data

- <http://devdata.worldbank.org/query/default.htm>
  - (1972-2004);
- <http://www.economicswbinstitute.org/concepts.htm>
  - (1986-2000);
- [http://www.oecd.org/about/0,2337,en\\_2649\\_201185\\_1\\_1\\_1\\_1\\_1,00.html](http://www.oecd.org/about/0,2337,en_2649_201185_1_1_1_1_1,00.html)
  - (2003-2004).

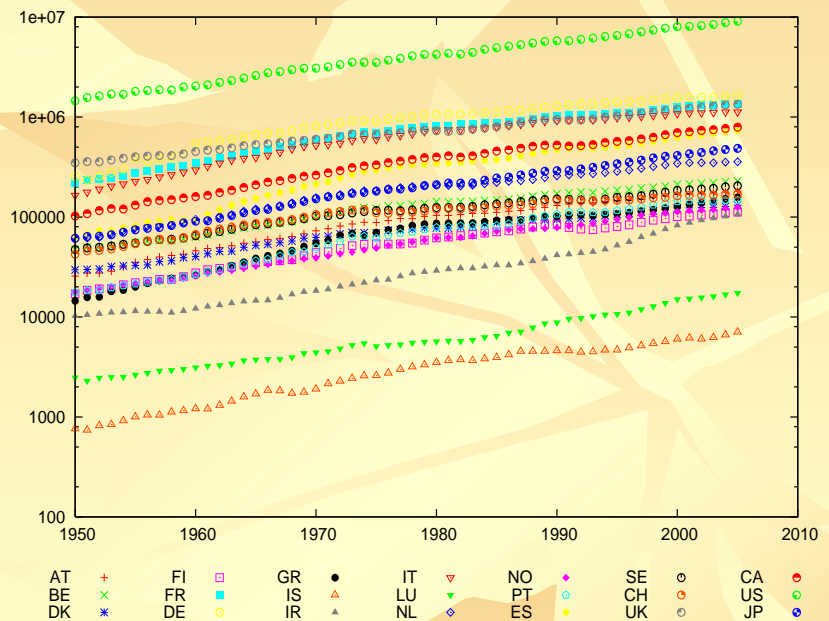
# GDP, C, K, B Data source

- <http://devdata.worldbank.org/query/default.htm>
- <http://www.cia.gov/cia/publications/factbook/rankorder/2004rank.html>
- <http://www.economicswbinstitute.org/concepts.htm>  
(data taken between 1986-2000);
- [http://www.oecd.org/about/0,2337,en\\_2649\\_201185\\_1\\_1\\_1\\_1\\_1,00.html](http://www.oecd.org/about/0,2337,en_2649_201185_1_1_1_1_1,00.html) (2003-2004).

# GDP data (EKS/GK)

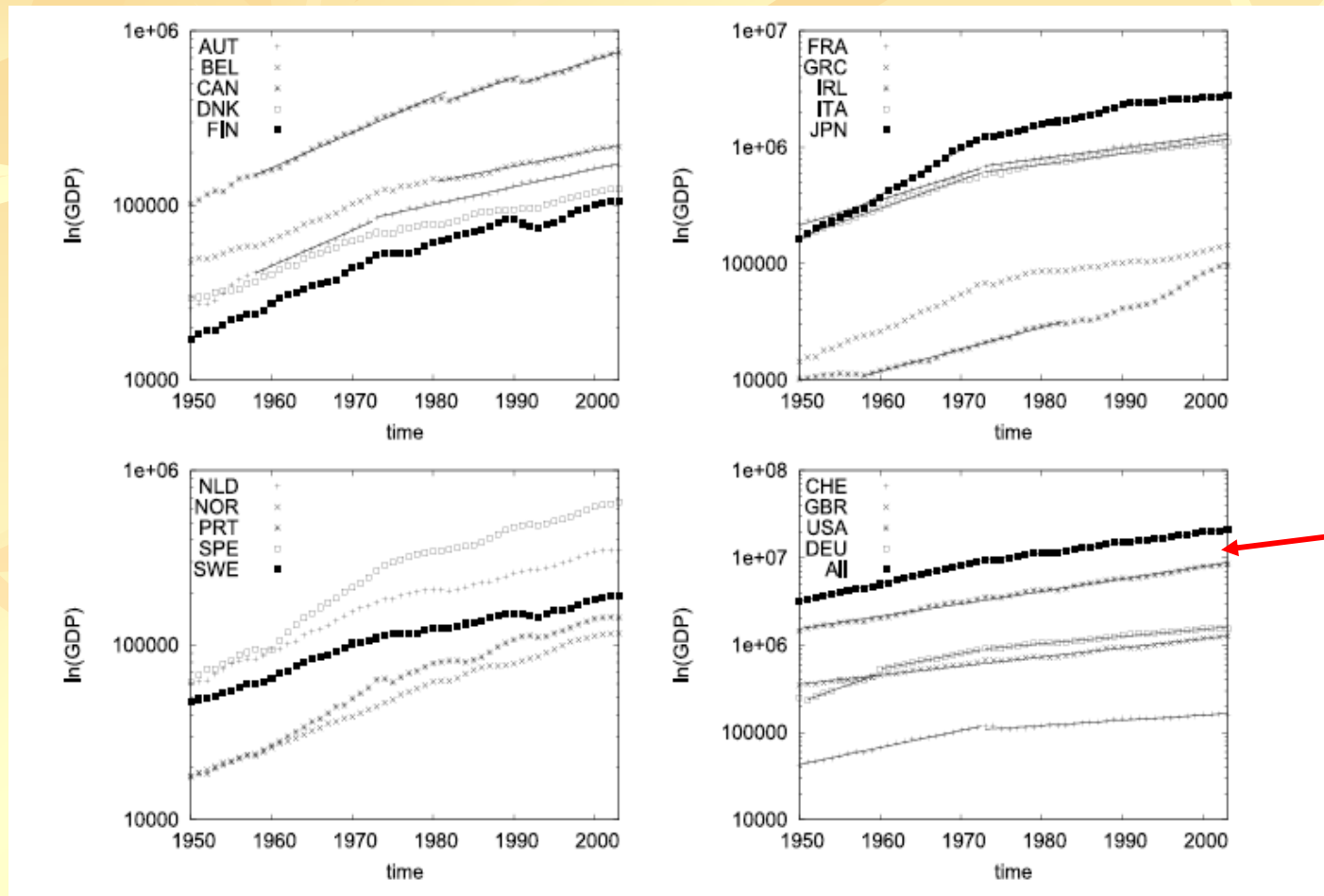


*GDP – EKS log*



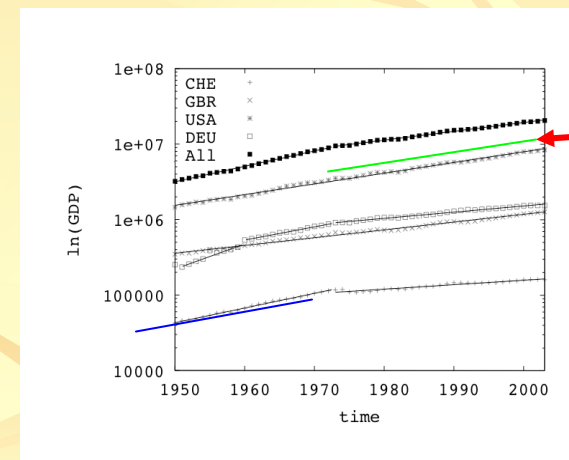
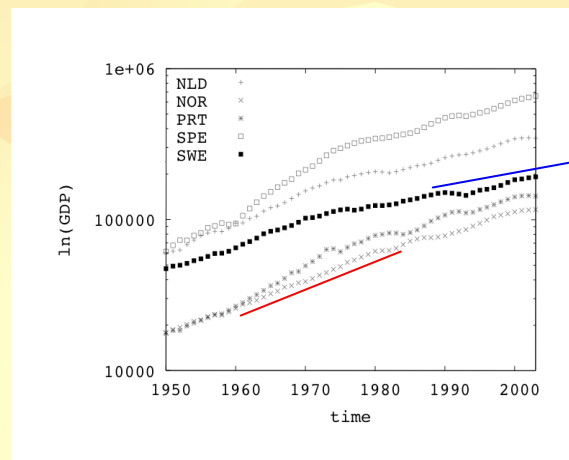
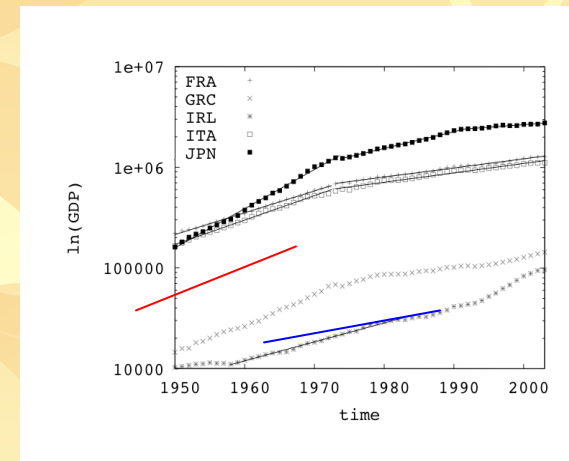
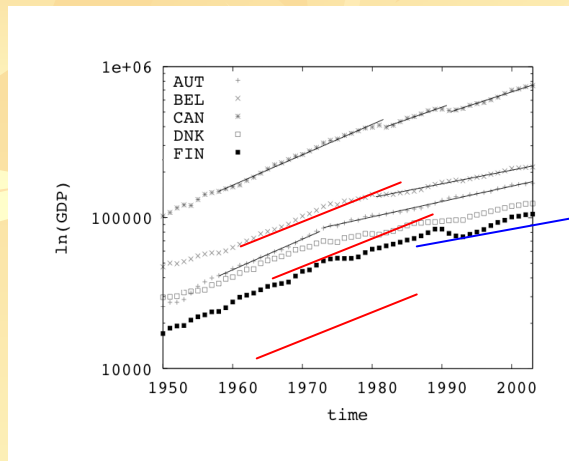
*GDP – GK log*

# GDP examples





# GDP “slopes”



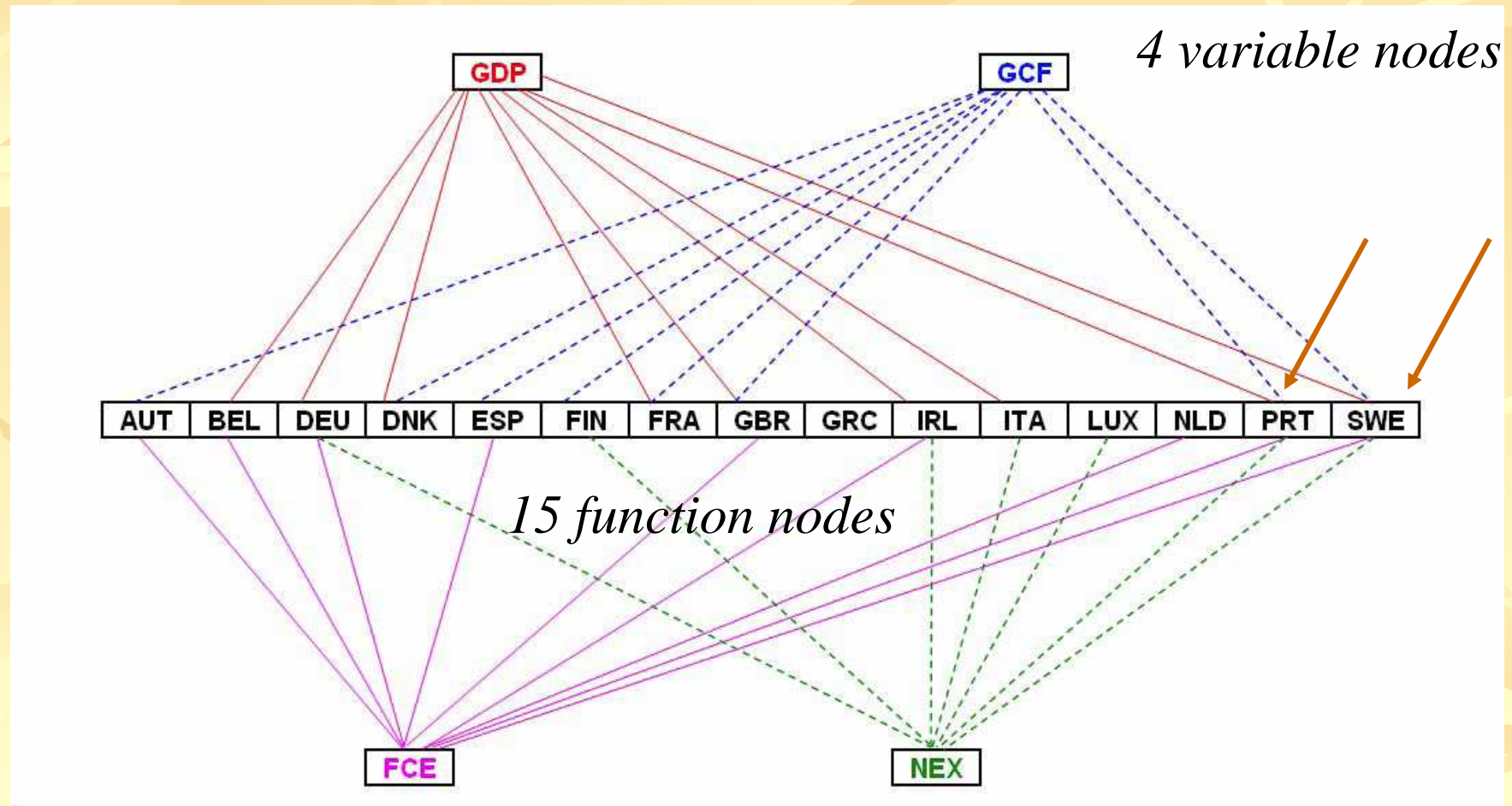
## 2. Methodology

- Cluster Variation Method (\*)
  - statistical mechanics and condensed matter
- Beyond a bipartite graph made of
  - (i) variables : vector nodes :  $i, j, \dots$  ,
    - macro-economic features, defining the phase space
  - (ii) function nodes :  $a, b, \dots$  .
    - countries , ... linked by (i)

(\*) A. Pelizzola

*Cluster Variation Method in Statistical Physics and Probabilistic Graphical Models*  
*J. Phys. A* **38** (2005) R309-339.

# Sort of “final” Cost Fcn. & Factor Graph



# “Economic Theory”

- **Hamiltonian = Cost function**

$$\mathcal{H} = \sum_a^N \mathcal{H}_a(s_a) =$$

$$= \mathcal{H}_0(s_0) + \mathcal{H}_1(s_1) + \mathcal{H}_2(s_2) + \dots$$

*N.B. Renormalization group philosophy*

# Hamiltonian “theory”

$$\mathcal{H} = \sum_a^N \mathcal{H}_a(s_a)$$

$s_a$        $a = 1, \dots (4)$

$$p(s) = \frac{1}{\mathcal{Z}} \exp[-\mathcal{H}(s)]$$

*Number of vector components*

$$\mathcal{Z} = \exp[-\mathcal{F}] = \sum_s \exp[-\mathcal{H}(s)]$$

*Number of « agents »*

$$\mathcal{S}_\alpha(s_\alpha) = - \sum_{s \in \alpha} p(s) \cdot \ln p(s)$$

$s_\alpha$        $\alpha = 1, \dots (M)$

### 3. Method : (1)

- A given macro economic indicator
  - e.g. Data : 1990 - 2005
- A set of countries; & *invent* “AVR” (“average”)
- Window for *t*-averaging : e.g.  $T = 5$  y or 10 y
- Correlations  $\Rightarrow$  distances
  - Adjacency matrix (\*)
- Network construction and evolution
  - According to some filter : correlation levels
  - $\Rightarrow$  communities
- (\*) Statistical distance matrix can also be constructed

# Correlations/distances (1)

$$C_{(t,T)}(A, B) = \frac{\langle AB \rangle_{(t,T)} - \langle A \rangle_{(t,T)} \langle B \rangle_{(t,T)}}{\sqrt{(\langle A^2 \rangle_{(t,T)} - \langle A \rangle_{(t,T)}^2)(\langle B^2 \rangle_{(t,T)} - \langle B \rangle_{(t,T)}^2)}}.$$

■  $A, B : MEI(t)$

$$d_s(A, B)_{(t,T)} = \sqrt{2(1 - C_{(t,T)}(A, B))}$$

$$C_{ij}(t, T) = \frac{\langle x_i x_j \rangle_T - \langle x_i \rangle_T \langle x_j \rangle_T}{\sqrt{\langle x_i^2 - \langle x_i \rangle_T^2 \rangle_T \langle x_j^2 - \langle x_j \rangle_T^2 \rangle_T}}$$



# Correlations/distances (2)

$$\hat{C}_{i,j}(k, T_2) = \left[ \frac{1}{\nu} \sum_{t=k}^{k+T_2} C_{i,j}^2(t, T_1) \right]^{1/2}$$

Usually:  $\nu = T$

■ But here:  $\nu = N - T + 1$

$$\hat{C}_{i,j}(T)$$

■ *N.B.*

■ “*T*”

■  $T_1$

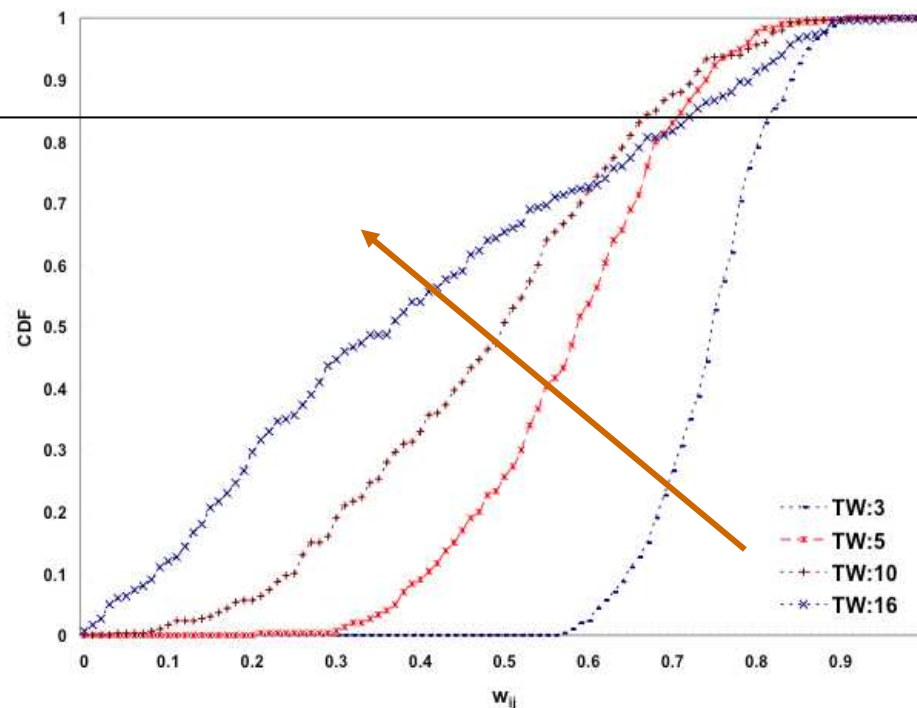
■  $T_2$

$$\hat{C}_{ij}(T) = \left[ \frac{1}{\nu} \sum_{t=k}^{k+T} C_{ij}^2(t) \right]^{1/2}$$

$$\hat{C}_{ii}(T) = \left[ \frac{1}{N - T + 1} \sum_{t=k}^{k+T} 1 \right]^{1/2} = \left[ \frac{T}{N - T + 1} \right]^{1/2}$$

$$d_{ii}(T) = \sqrt{2(1 - \hat{C}_{ii}(T))}.$$

# Cumulative distribution function



$$M = 25$$

*For  
different  
time  
window  
sizes*

$$w_{ij}(T) \equiv \hat{C}_{ij}(T)$$

### 3. Method : (2)

- A given macro economic indicator
  - e.g. Data : 1990 - 2005
- A set of countries; *invent* “ALL” (“average”)
- Window for *t*-averaging : e.g.  $T = 5$  y or 10 y
- Correlations  $\Rightarrow$  distances
  - Adjacency matrix
- Network construction and evolution
  - According to some filter : correlation levels
  - $\Rightarrow$  communities
- Statistical distance matrix can also be constructed

$$w_{ij}(T) \equiv \hat{C}_{ij}(T)$$

# Networks (1)

- Complete Graphs
- Minimum Spanning Tree
- Minimal Length Path
  - UMLP
  - BLMP

# Networks (2)

- **UMLP**

- *Attachment starting from seed (« ALL »)*
- *Linear ; Unidirectional*

- **BLMP**

- *Attachment starting from seed (« ALL »)*
- *Linear ; Bidirectional*

- **LMST**

- *root : the pair of closest neighbouring countries*
- *Then the country closest to any node is searched for and attached.*

# Network characteristics (1)

## *Degree distribution*

*Vertex degree (in weighted network):*

$$k_i = \sum_{j=1, j \neq i}^M w_{ij}$$

*(T)*

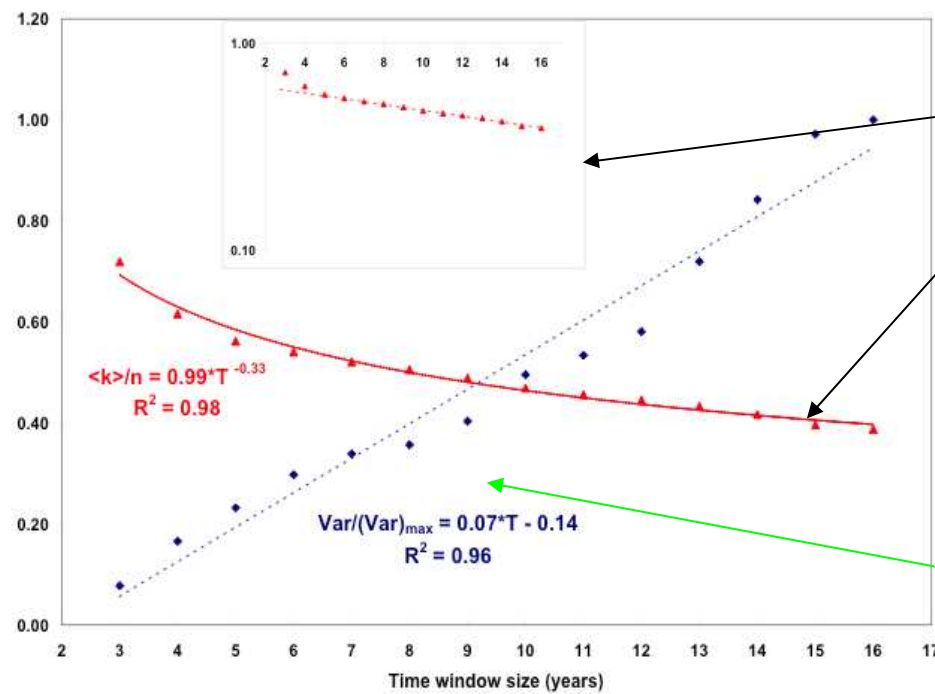
*thus:*

$$\langle k \rangle = \frac{1}{M} \sum_{i=1, i \neq j}^M \sum_{j=1}^M w_{ij}$$

# Average degree

$$\frac{\langle k \rangle}{M}$$

$$\langle k \rangle \sim \frac{1}{T^{1/3}}$$



*EU25*



# Network characteristics (2)

## *Overlap coefficient*

*in weighted networks:*

*Number of common neighbors*

$$N_{ij} = \sum_{l=1}^M (w_{il} + w_{jl})$$

*Overlapping coefficient*

$$O_{ij} = \frac{N_{ij}}{4(M-1)(M-2)} \left[ \sum_{p=1; p \neq i}^M w_{ip} + \sum_{q=1; q \neq j}^M w_{jq} \right]$$

*which generalizes:*

$$O_{ij} = \frac{N_{ij}(k_i + k_j)}{4(M-1)(M-2)}, \quad i \neq j.$$

# Network characteristics (AOI)

Country *Averaged Overlapping coefficient*

EU25

(90-05)

( $T=5$ )

$$\langle O_i \rangle = \frac{1}{M-1} \sum_{j=1}^M O_{ij}$$

	$\langle O_i \rangle$		$\langle O_i \rangle$		$\langle O_i \rangle$		$\langle O_i \rangle$		$\langle O_i \rangle$
SWE	0.38	SVK	0.37	AUT	0.35	POL	0.34	LTU	0.32
GER	0.37	BEL	0.36	FIN	0.35	MLT	0.33	LVA	0.31
FRA	0.37	IRL	0.36	PRT	0.35	GRC	0.33	CZE	0.31
DNK	0.37	LUX	0.36	NLD	0.35	CYP	0.32	EST	0.30
HUN	0.37	ESP	0.35	ITA	0.35	SVN	0.32	GBR	0.29

PRE

# AOI hierarchy

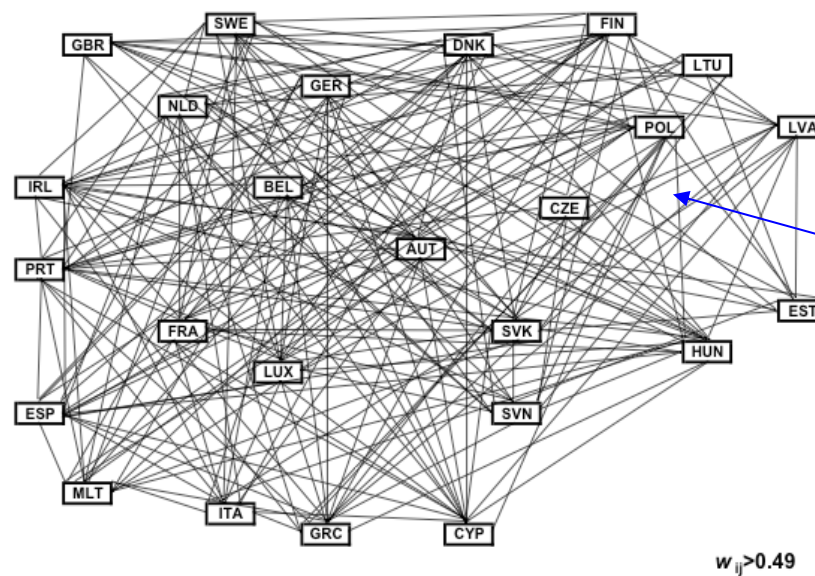
*EU25*

25 European countries, ranked according to their AOI, from GDP/capita yearly data between 1990 and 2005. The Income group according to the World Bank ATLAS method, *t*-Student confidence interval (*tci*) for the AOI and  $k_i$  are given

Country Name	Acronym	Income Group	AOI	( <i>tci</i> )	$k_i$
Ireland	IRL	OEI	0.800	(0.799; 0.801)	21.878
Austria	AUT	OEI	0.799	(0.798; 0.800)	21.853
Denmark	DNK	OEI	0.798	(0.797; 0.799)	21.825
Finland	FIN	OEI	0.798	(0.797; 0.799)	21.812
Spain	ESP	OEI	0.796	(0.795; 0.797)	21.775
Belgium	BEL	OEI	0.796	(0.795; 0.797)	21.769
Poland	POL	UMI	0.794	(0.792; 0.795)	21.698
Hungary	HUN	OEI	0.792	(0.791; 0.793)	21.664
Slovenia	SVN	NOI	0.792	(0.791; 0.793)	21.644
Sweden	SWE	OEI	0.787	(0.786; 0.788)	21.509
United Kingdom	GBR	OEI	0.784	(0.783; 0.785)	21.431
France	FRA	OEI	0.777	(0.776; 0.778)	21.232
Estonia	EST	NOI	0.774	(0.773; 0.775)	21.155
Luxembourg	LUX	OEI	0.766	(0.765; 0.767)	20.945
Malta	MLT	NOI	0.766	(0.765; 0.767)	20.934
Slovak Republic	SVK	OEI	0.765	(0.764; 0.766)	20.905
Italy	ITA	OEI	0.764	(0.763; 0.765)	20.870
Germany	DEU	OEI	0.763	(0.762; 0.765)	20.864
Greece	GRC	OEI	0.761	(0.760; 0.762)	20.804
Netherlands	NLD	OEI	0.754	(0.753; 0.755)	20.603
Lithuania	LTV	UMI	0.730	(0.729; 0.731)	19.919
Portugal	POR	OEI	0.719	(0.718; 0.720)	19.590
Czech Republic	CZE	OEI	0.717	(0.716; 0.717)	19.529
Switzerland	CHE	OEI	0.678	(0.677; 0.679)	18.401
mean.			0.769	(0.768; 0.770)	

# Percolation point of view (1)

$$n = T = 5; t = 0.98$$

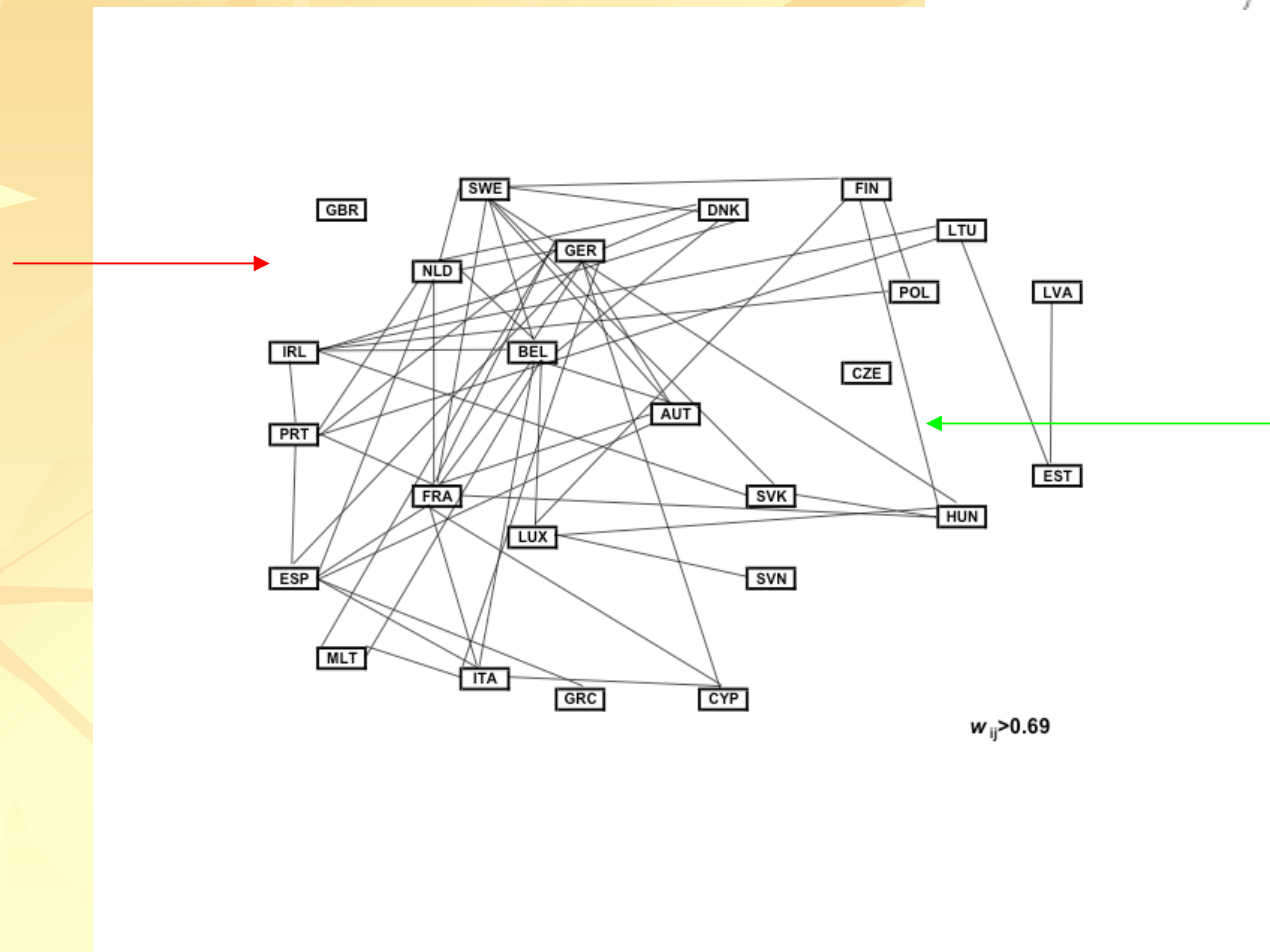


**EU25**

60%

# Percolation point of view (2)

$$n = T = 5; t = 1.64$$

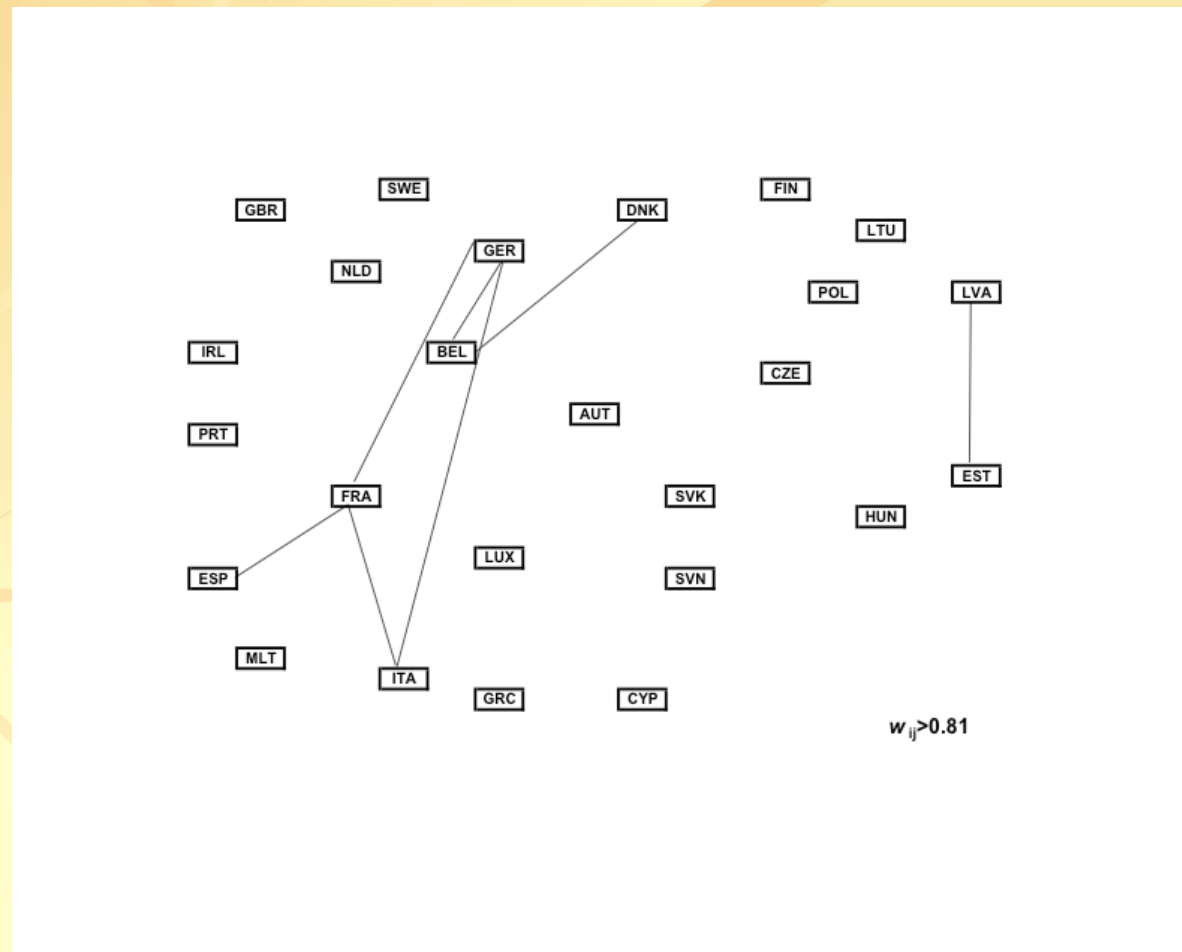


EU25

80%

# Percolation point of view (3)

$$n = T = 5; t = 2.35$$

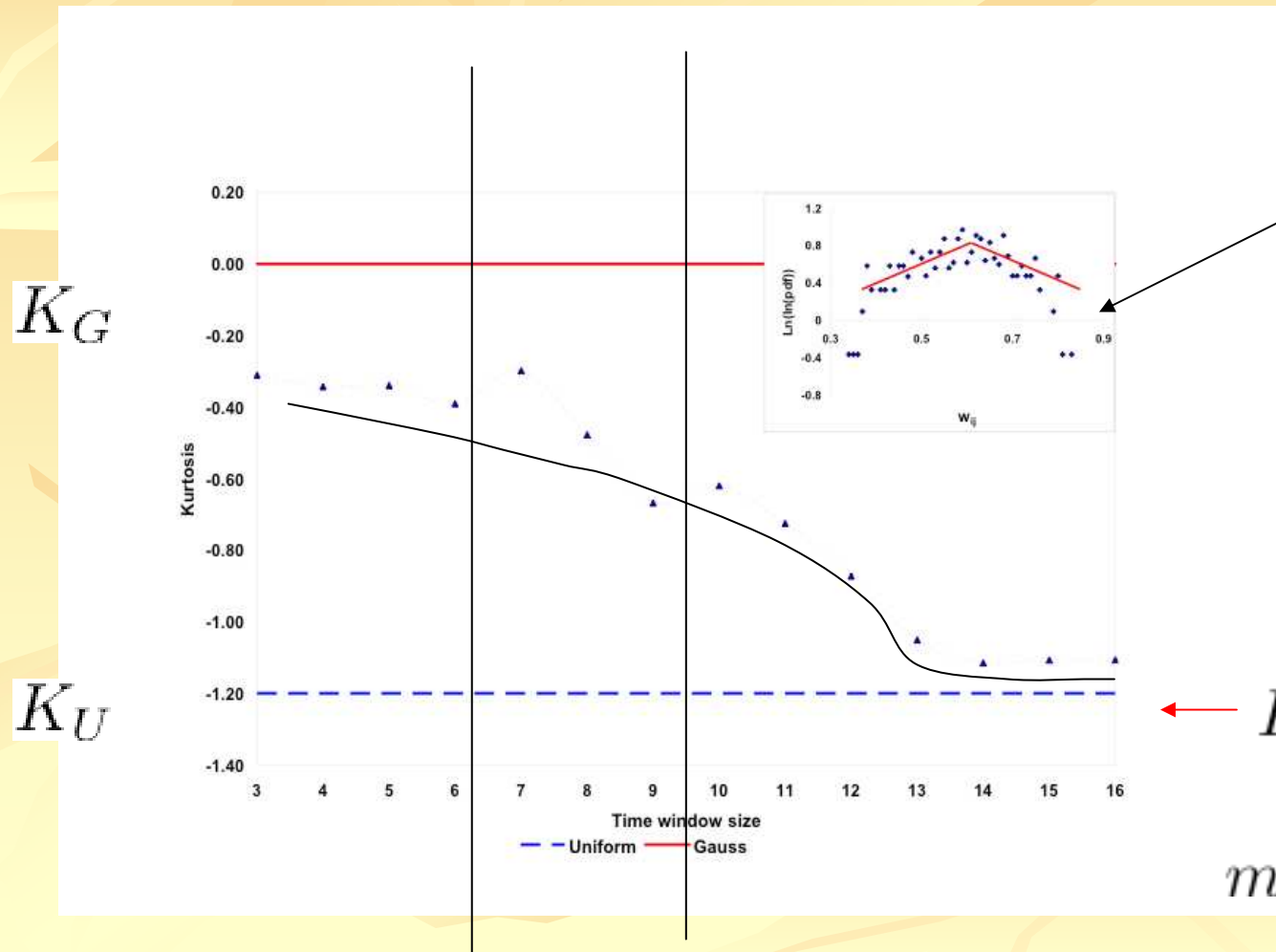


EU25

90%

$w_{ij} > 0.81$

# Kurtosis vs. window size



$$K_U = -\frac{6m^2 + 1}{5m^2 - 1}$$

$$m = 25 * 12 = 300$$



# Properties of $O_{ij}$

- *Fully disconnected :*

$$O_{ij} = 0$$

- *Fully connected*

$$O_{ij} = 1$$

$$(N_{ij} = M - 2; k_i = k_j = M - 1)$$

- *Otherwise ....*

- *“average”, for a given node :*

$$\langle O_i \rangle = \frac{1}{M-1} \sum_{m=1}^M O_{im}$$

$\Rightarrow$  *hierarchy*

# 25+18+1 AOI,... (90-05)

( $T=5$ )

Country	Acronym	AOI (tci)	$k_i$	$L_i$	$E_i$
Ireland	IRL	0.709 (0.705; 0.712)	36.78		
Denmark	DNK	0.705 (0.702; 0.708)	36.61		
Finland	FIN	0.700 (0.697; 0.703)	36.35		
Austria	AUT	0.700 (0.696; 0.703)	36.33		
Poland	POL	0.697 (0.694; 0.701)	36.22		
Slovenia	SVN	0.696 (0.693; 0.699)	36.16		
Belgium	BEL	0.695 (0.692; 0.699)	36.11		
Hungary	HUN	0.694 (0.690; 0.697)	36.02		
Sweden	SWE	0.693 (0.690; 0.696)	36.00		
United States	USA	0.692 (0.689; 0.695)	35.94		
United Kingdom	GBR	0.690 ( 0.688; 0.691 )	35.86		
Spain	ESP	0.689 (0.686; 0.692)	35.79		
Luxembourg	LUX	0.685 (0.682; 0.688)	35.57		
Malta	MLT	0.679 (0.676; 0.682)	35.26		
Latvia	LVA	0.6894 (0.6881; 0.6908)	35.43		
Estonia	EST	0.678 (0.675; 0.681)	35.24		
France	FRA	0.673 (0.670; 0.677)	34.99		
Nicaragua	NIC	0.673 (0.670; 0.676)	34.96		
Guatemala	GTM	0.672 (0.669; 0.675)	34.92		
Slovak Rep.	SVK	0.672 (0.669; 0.675)	34.91		
Netherlands	NLD	0.668 (0.665; 0.671)	34.72		
Germany	DEU	0.665 (0.662; 0.668)	34.55		
Chile	CHL	0.664 (0.661; 0.667)	34.48		

# 18 LA + USA

18 Latin American countries, ranked according to their AOI, from GDP/capita yearly data between 1990 and 2005. Income group according to the World Bank ATLAS method,  $t$ -Student confidence interval ( $tci$ ) for the AOI and  $k_i$  are given

Country Name	Acronym	Income Group	AOI ( $tci$ )	$k_i$
Colombia	COL	LMC	0.594 (0.591; 0.597)	13,556
El Salvador	SLV	LMC	0.590 (0.587; 0.593)	13,454
Guatemala	GTM	LMC	0.584 (0.581; 0.587)	13,335
Chile	CHL	UMC	0.577 (0.574; 0.581)	13,176
Nicaragua	NIC	LMC	0.576 (0.573; 0.579)	13,141
Bolivia	BOL	LMC	0.571 (0.568; 0.574)	13,023
Costa Rica	CRI	UMC	0.567 (0.564; 0.570)	12,935
Panama	PAN	UMC	0.565 (0.561; 0.568)	12,880
Dominican Rep.	DOM	LMC	0.557 (0.554; 0.560)	12,713
Uruguay	URY	UMC	0.556 (0.553; 0.558)	12,673
Paraguay	PRY	LMC	0.543 (0.541; 0.546)	12,380
Argentina	ARG	UMC	0.536 (0.534; 0.539)	12,211
Honduras	HND	LMC	0.534 (0.532; 0.537)	12,170
Ecuador	ECU	LMC	0.532 (0.530; 0.535)	12,122
Mexico	MEX	UMC	0.531 (0.528; 0.534)	12,091
Peru	PER	UMC	0.527 (0.524; 0.530)	11,985
Brazil	BRA	UMC	0.526 (0.523; 0.530)	11,978
Venezuela R.B.	VEN	UMC	0.502 (0.500; 0.504)	11,383
mean.			0.553 (0.551; 0.555)	

18 Latin American countries and USA, ranked according to their AOI, from GDP/capita yearly data between 1990 and 2005. Income group according to the World Bank ATLAS method,  $t$ -Student confidence interval ( $tci$ ) for the AOI and  $k_i$  are given

Country Name	Acronym	Income Group	AOI ( $tci$ )	$k_i$
United States	USA	OEI	0.620 (0.617; 0.623)	14,814
Colombia	COL	LMC	0.605 (0.602; 0.608)	14,454
El Salvador	SLV	LMC	0.601 (0.598; 0.604)	14,351
Guatemala	GTM	LMC	0.596 (0.593; 0.599)	14,245
Chile	CHL	UMC	0.588 (0.584; 0.591)	14,040
Nicaragua	NIC	LMC	0.587 (0.584; 0.590)	14,024
Bolivia	BOL	LMC	0.581 (0.578; 0.584)	13,887
Costa Rica	CRI	UMC	0.578 (0.575; 0.581)	13,804
Panama	PAN	UMC	0.575 (0.572; 0.579)	13,745
Dominican Rep.	DOM	LMC	0.569 (0.566; 0.572)	13,584
Uruguay	URY	UMC	0.564 (0.562; 0.567)	13,472
Paraguay	PRY	LMC	0.554 (0.552; 0.556)	13,218
Argentina	ARG	UMC	0.543 (0.541; 0.546)	12,952
Honduras	HND	LMC	0.543 (0.540; 0.545)	12,932
Ecuador	ECU	LMC	0.541 (0.538; 0.544)	12,895
Mexico	MEX	UMC	0.541 (0.538; 0.544)	12,890
Peru	PER	UMC	0.535 (0.532; 0.538)	12,746
Brazil	BRA	UMC	0.535 (0.532; 0.538)	12,739
Venezuela R.B.	VEN	UMC	0.508 (0.505; 0.510)	12,041
mean.			0.586 (0.584; 0.588)	

# Network characteristics (3)

## *Global /local path length*

Characteristic path length

$$L = \frac{1}{N(N-1)} \sum_{i,j \in N; i \neq j} d_{ij}$$

$$d_{ij}(T) = \sqrt{2(1 - \hat{C}_{ij}(T))}$$

local path length  $L_i$  attributed to each node, e.g. for node  $i$

$$L_i = \frac{1}{(N-1)} \sum_{j \in N} d_{ij}$$

# Network characteristics (4)

## *Global /local efficiency*

Characteristic global efficiency

$$E = \frac{1}{N(N-1)} \sum_{i,j \in N; i \neq j} \frac{1}{d_{ij}}$$

local efficiency  $E_i$  for node  $i$ , i.e.

$$E_i = \frac{1}{(N-1)} \sum_{j \in N} \frac{1}{d_{ij}}$$

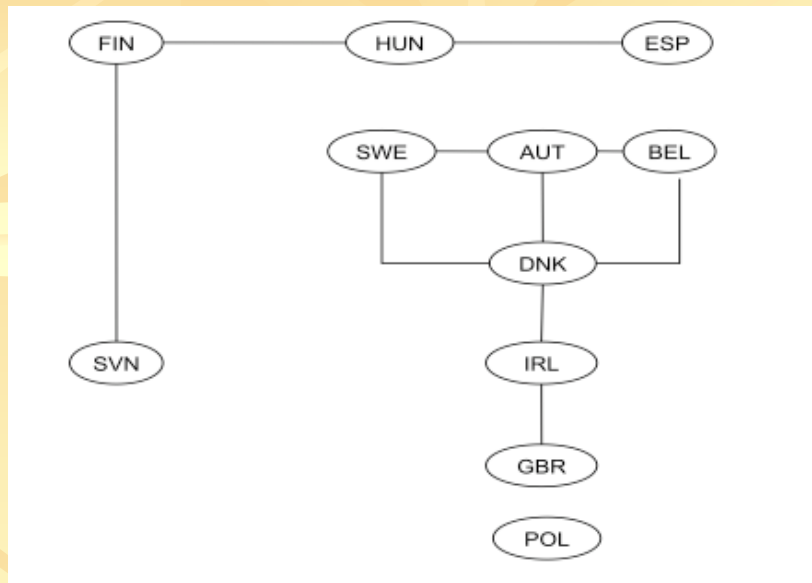
# *GNI/c (1)*

*2007*

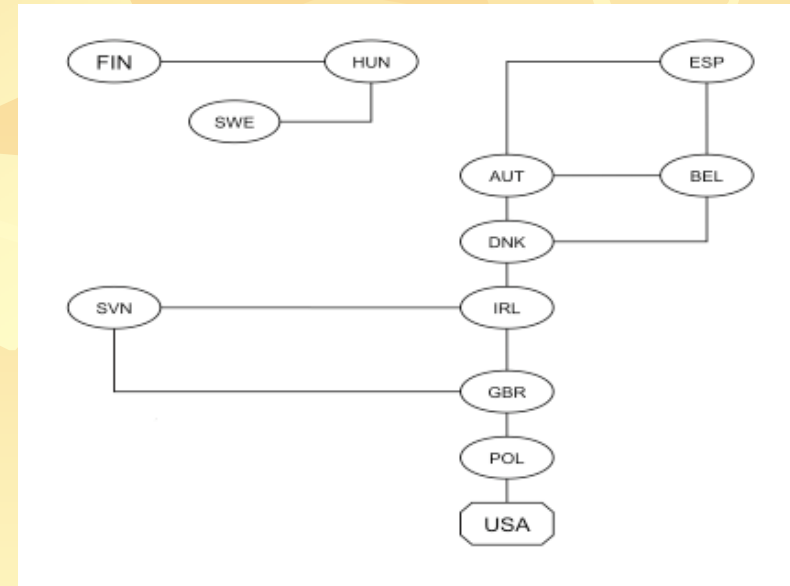
- low income (LC), if the income per capita is USD 935 or less
- lower middle income (LMC), USD 936 - 3.705
- upper middle income (UMC), USD 3.706 - 11.455;
- high income (HC), USD 11.456 or more.



# EU25 vs. LA18-EU25-USA



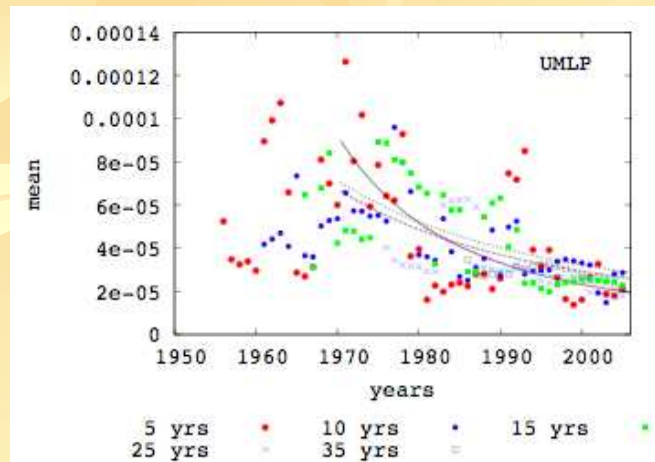
$AOI > 0.686$



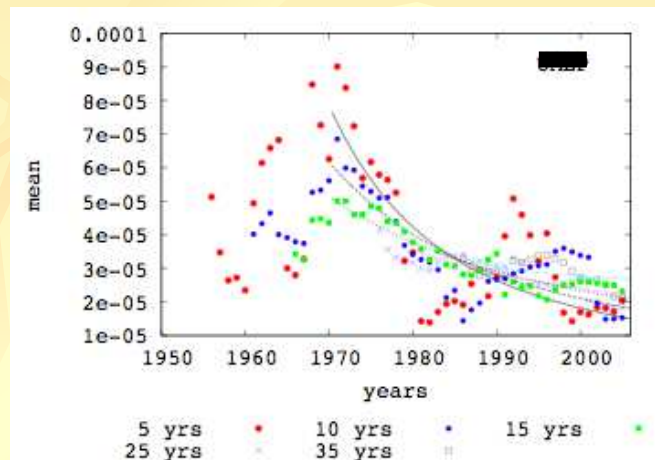
0.784



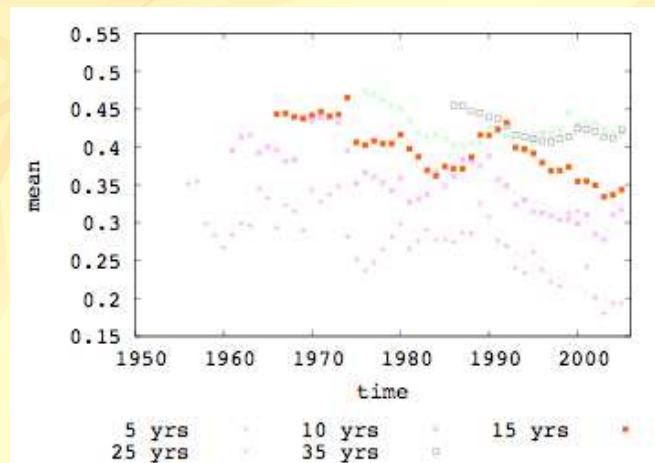
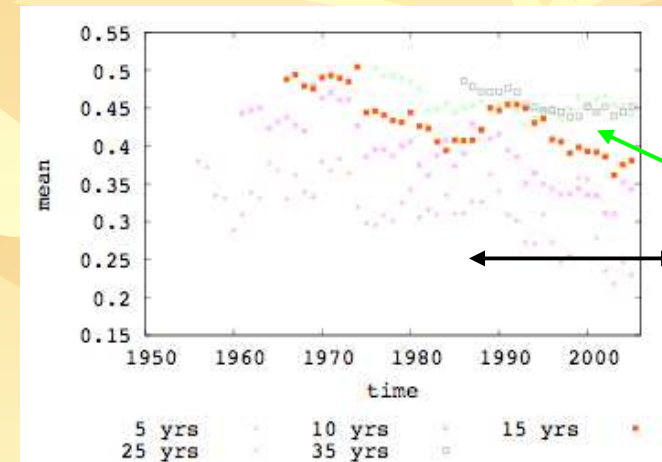
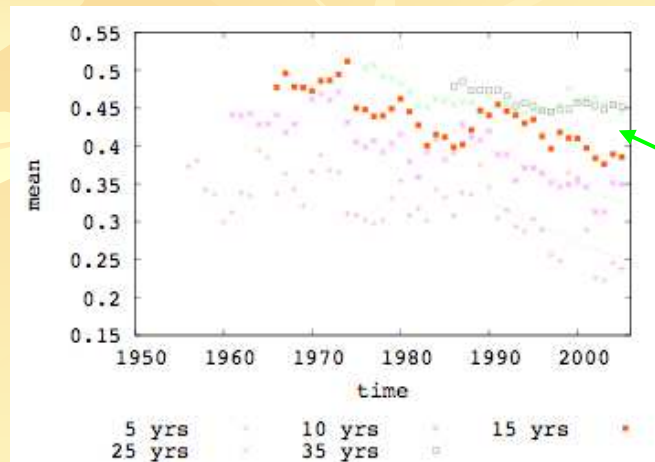
# Mean statistical distance



$$d_s(A, B)_{(t,T)} = \sqrt{2(1 - C_{(t,T)}(A, B))}$$



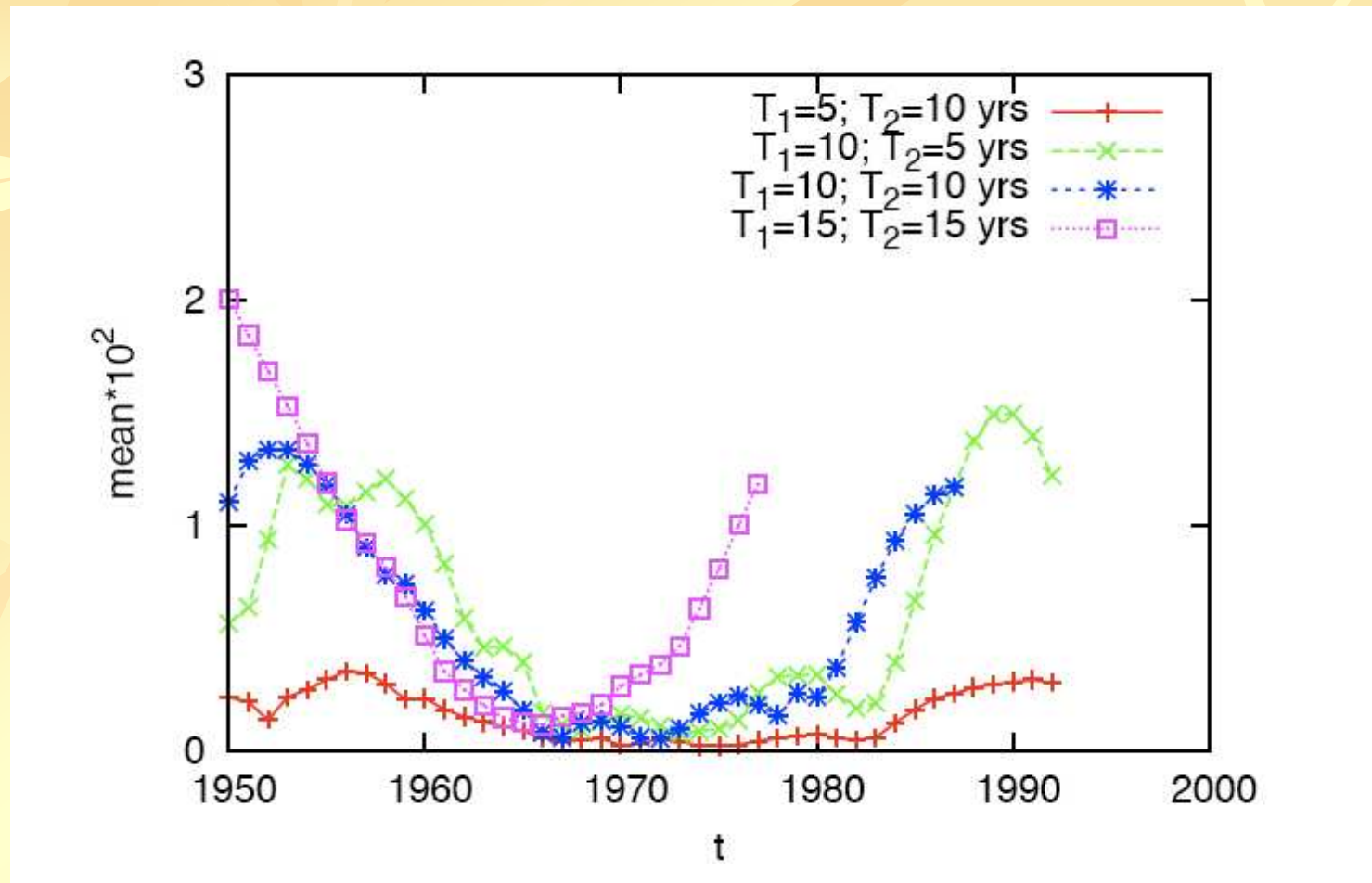
# Usual « statistical distance » (Theil index) Distance correlations



$$d_s(A, B)_{(t,T)} = \sqrt{2(1 - C_{(t,T)}(A, B))}$$

3 different types of  
networks :  
=> Somewhat network  
independent

# Mean distance



## Conclusion (2) : *globalization*

- *Observed globalization process does not depend on the type of studied network*
- *« No » globalization process ca. 1960*
  - *as seen through mean distance maximum*
- *Globalization process since 1970 and destabilisation after 2000*
  - *as observed as a decrease of the network size.*
- *Globalization process is better seen if time lag greater than 5 yrs.*
- *Shift of the maximum is consistent with increase in time lag.*
- *q-Theil distance analysis gives more consistent results*

### 3. Method : (3)

- A given macro economic indicator
  - e.g. Data : 1990 - 2005
- A set of countries; *invent* “AVR” (“average”)
- Window for *t*-averaging : e.g.  $T = 5$  y or 10 y
- Correlations  $\Rightarrow$  distances
  - Adjacency matrix
- Network construction and evolution
  - According to some filter : correlation levels
  - $\Rightarrow$  communities
- Statistical distance matrix can also be constructed

## 4. MA-MPL (1)

- Repeat for each macro economic indicator
  - e.g. Data : 1990 - 2005
- same countries; same windows for  $t$ -averaging
- Correlation matrices (4)  $\Rightarrow$
- Statistical distance matrices (4)
- Networks
  - According to some filter : correlation level



# MA-MLP (2)

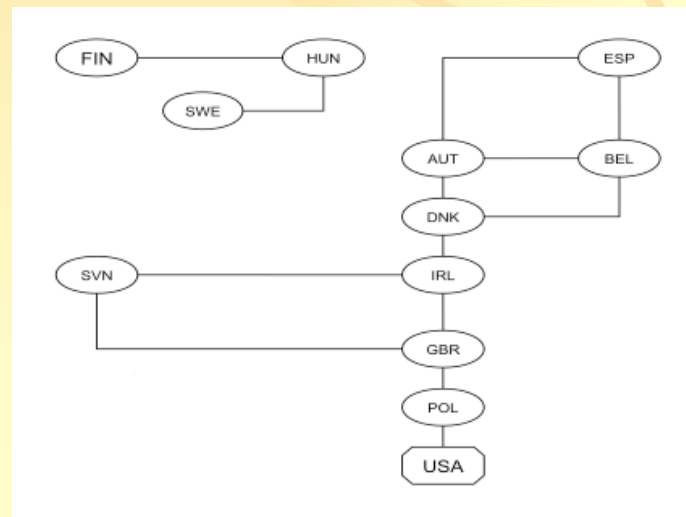
- Compare correlations
  - Distance correlations =>
  - Clusters (def. next)
- Movements inside hierarchy and networks
  - Statistics : searching for “stability” of clusters
    - Resulting from “average” (over indices)
    - “sensitivity degree”
- => Hamiltonian or Cost function
  - Entropy ...



# Cluster def.

- A cluster is a subset of the factor graph such that
  - if a “function node” belongs to the cluster
  - all “variable nodes” belong to the cluster
    - “function node” : country
    - “variable node” : MEindex

*e.g.* GDP/c : EU25&USA:



# Clusters

- *Group countries according to weights, links*
- *For various time windows*
- *Moved as a function of time*
- *Look for consistency through threshold*
- *At “confidence level”*

# 5. Movement correlations

$$w_{ij}$$

$$C_{ij} \Rightarrow \hat{C}_{ij}$$

$$d_{ij}$$

$$CDF(d_{ij})$$

$$L_i$$

$$\langle L_i \rangle - L_i = \hat{d}_i$$


$$\hat{C}_{ij}(t) = \frac{\langle \hat{d}_i(t) \hat{d}_j(t) \rangle - \langle \hat{d}_i(t) \rangle \langle \hat{d}_j(t) \rangle}{\sqrt{\langle [\hat{d}_i(t)]^2 \rangle - \langle \hat{d}_i(t) \rangle^2} \sqrt{\langle [\hat{d}_j(t)]^2 \rangle - \langle \hat{d}_j(t) \rangle^2}}$$

- $\hat{d}_i(t)$  : minimal path length distance to the average
- $C > 0.9$  ;  $C < -0.5$
- [FRA-SWE-DEU] & [BEL-GBR-IRE-DNK-PRT]
- ITA; LUX; AUT; GRC


# $(s_1)$ GDP MPL $d$ to AVR 1

	AT	BE	DE	DK	ES	FI	FR	UK	GR	IE	IT	LU	NL	PT	SE
94-98	.67	.86	.86	.86	.40	.40	.67	.86	.40	.86	.86	.40	.40	.86	.86
95-99	.60	.65	.52	.71	.21	.77	.45	.77	.37	.65	.90	.37	.23	.83	.52
96-00	.58	.32	.46	.61	.34	.81	.46	.32	.32	.53	.32	.20	.60	.60	.46
97-01	.48	.30	.48	.30	.28	.42	.48	.44	.68	.38	.68	.14	.28	.28	.48
98-02	.43	.26	.19	.19	.21	.43	.19	.19	1.04	.29	.44	.12	.21	.21	.29
99-03	.25	.23	.19	.19	.29	.26	.19	.37	1.15	.26	.37	.23	.19	.19	.28
00-04	.27	.27	.17	.26	.28	.27	.21	.27	.53	.50	.28	.27	.21	.21	.27

# $(s_1)$ GDP Corr. 2



	AT	BE	DE	DK	ES	FI	FR	UK	GR	IE	IT	LU	NL	PT	SE
AT	1	.77	.88	.88	.33	.69	.88	.69	-.69	.75	.71	.42	.61	.89	.85
BE		1	.88	<u>.90</u>	.41	.27	.80	<u>.94</u>	-.59	<u>.92</u>	.83	.85	.23	<u>.90</u>	.91
DE			1	.90	.61	.35	<u>.98</u>	.86	-.65	.85	.78	.61	.52	.86	.99
DK				1	.50	.58	.87	.84	-.80	<u>.93</u>	.67	.77	.58	<u>.99</u>	<u>.88</u>
ES					1	-.10	.61	.34	-.38	<u>.55</u>	→ .05	.36	.66	<u>.37</u>	.64
FI						1	.42	.25	-.62	.34	.27	.14	.60	.64	.26
FR							1	.79	-.71	.81	.73	.52	.60	.82	.95
UK								1	-.52	.82	.90	.85	.12	.86	.86
GR									1	-.82	-.38	-.56	-.62	-.76	-.60
IE										1	.63	.85	.43	.89	.87
IT											1	→ .59	→ -.05	.73	.77
LU												1	.06	.77	.65
NL													1	.50	.47
PT														1	.84
SE															1



# $(s_2)$ FCE MPL $d$ to AVR<sub>2</sub>

	AT	BE	DE	DK	ES	FI	FR	UK	GR	IE	IT	LU	NL	PT	SE
94-98	.88	.65	.85	.88	.65	.37	.65	.65	.65	.65	.37	.65	.65	.65	.65
95-99	.79	.79	.79	.81	.79	.41	.79	.79	.93	.79	.53	.59	.79	.79	.79
96-00	1.02	1.02	1.02	1.02	1.02	1.02	1.02	1.02	1.02	1.02	.26	1.02	1.02	1.02	1.02
97-01	.51	.51	.51	.65	.51	.73	.88	.51	.65	.51	.33	.88	.51	.51	.51
98-02	.52	.52	.52	.96	.52	.66	.95	.65	.96	.52	.35	<u>1.19</u>	.52	.52	.52
99-03	.45	.42	.45	1.00	.45	.53	.40	.46	1.00	.42	.30	.92	.45	.45	.45
00-04	.88	.65	.85	.88	.65	.37	.65	.65	.65	.65	.37	.65	.65	.65	.65

# Movement correlations inside ( $s_2$ ) (FCE) hierarchy

CORRELATIONS BETWEEN THE MOVEMENTS OF COUNTRIES INSIDE HIERARCHY


	AUT	BEL	DEU	DNK	ESP	FIN	FRA	GBR	GRC	IRL	ITA	LUX	NLD	PRT
AUT	1.00	0.92	1.00	0.23	0.92	0.21	0.38	0.87	0.03	0.92	0.07	-0.34	0.92	0.92
BEL		1.00	0.94	0.23	1.00	0.45	0.56	0.97	0.28	1.00	0.06	-0.15	1.00	1.00
DEU			1.00	0.24	0.93	0.24	0.40	0.89	0.07	0.94	0.07	-0.32	0.93	0.93
DNK				1.00	0.26	0.22	-0.14	0.35	0.75	0.23	-0.41	0.44	0.26	0.26
ESP					1.00	0.45	0.53	0.97	0.31	1.00	0.04	-0.15	1.00	1.00
FIN						1.00	0.65	0.49	0.34	0.45	-0.68	0.68	0.45	0.45
FRA							1.00	0.64	0.05	0.56	-0.05	0.38	0.53	0.53
GBR								1.00	0.40	0.97	0.03	0.02	0.97	0.97
GRC									1.00	0.28	-0.11	0.45	0.31	0.31
IRL										1.00	0.06	-0.15	1.00	1.00
ITA											1.00	-0.68	0.04	0.04
LUX												1.00	-0.15	-0.15
NLD													1.00	1.00
PRT														1.00
SWE														



## (s<sub>2</sub>) FCE Corr. 3


	AT	BE	DE	DK	ES	FI	FR	UK	GR	IE	IT	LU	NL	PT	SE
AT	1	.92	1	.23	.92	.21	.38	.87	.03	.92	.07	-.34	.92	.92	.92
BE		1	.94	.23	1	.45	.56	.97	.28	1	.06	-.15	1	1	1
DE			1	.24	.93	.24	.40	.89	.07	.94	.07	-.32	.93	.93	.93
DK				1	.26	.22	-.14	.35	.75	.23	-.41	.44	.26	.26	.26
ES					1	.45	.53	.97	.31	1	.04	-.15	1	1	1
FI						1	.65	.49	.34	.45	-.68	.68	.45	.45	.45
FR							1	.64	.05	.56	-.05	.38	.53	.53	.53
UK								1	.40	.97	.03	.02	.97	.97	.97
GR									1	.28	-.11	.45	.31	.31	.31
IE										1	.06	-.15	1	1	1
IT											1	-.68	.04	.04	.04
LU												1	-.15	-.15	-.15
NL													1	1	1
PT														1	1
SE															1

# $(s_3)$ GCF MPL $d$ to AVR 4



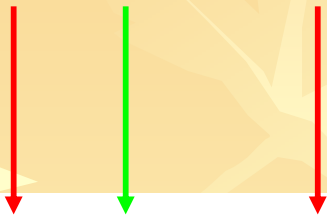
	AT	BE	DE	DK	ES	FI	FR	UK	GR	IE	IT	LU	NL	PT	SE
94-98	.51	.48	.59	.52	.66	.48	.66	.58	.89	.67	.38	.85	.67	.37	.51
95-99	.47	.46	.75	.49	.54	.46	.54	.61	.75	.49	.33	.83	.49	.39	.58
96-00	.75	.78	.75	.78	.75	.78	.75	.58	.75	.84	.32	.32	.48	.20	.75
97-01	.70	.47	.70	.62	.70	.62	.70	.57	.70	.38	.63	.29	.29	.09	.70
98-02	.46	.46	.46	.68	.46	.68	.46	.61	.46	.46	1.13	.46	.46	.46	.46
99-03	.70	.70	.70	.88	.70	.88	.70	.70	.70	.70	1.07	.70	.70	.70	.70

# $(s_3)$ GCF Corr. 5




	AT	BE	DE	DK	ES	FI	FR	UK	GR	IE	IT	LU	NL	PT	SE
AT	1	.76	.59	.68	.88	.69	.88	.10	.19	.45	-.04	-.58	-.12	-.26	.94
BE		1	.47	.81	.67	.79	.67	.35	.15	.85	-.02	-.27	.32	.15	.73
DE			1	.10	.64	.09	.64	.05	.55	.30	-.57	-.02	-.08	-.25	.81
DK				1	.41	1	.41	.61	-.32	.50	.56	-.40	.24	.39	.55
ES					1	.40	1	-.04	.61	.58	-.35	-.26	.11	-.29	.83
FI						1	.40	.58	-.37	.46	.57	-.46	.17	.35	.56
FR							1	-.04	.61	.58	-.35	-.26	.11	-.29	.83
UK								1	-.21	.20	.63	.37	.61	.91	.12
GR									1	.44	-.76	.45	.37	-.20	.27
IE										1	-.26	.10	.62	.21	.40
IT											1	-.15	.12	.60	-.21
LU												1	.73	.60	-.46
NL													1	.78	-.17
PT														1	-.27
SE															1

# $(s_4)$ NEX MPL $d$ to AVR 6


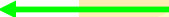
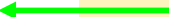


	AT	BE	DE	DK	ES	FI	FR	UK	GR	IE	IT	LU	NL	PT	SE
94-98	1.27	.19	.65	.89	.45	.80	.65	.62	.75	.62	.62	.80	.64	.62	.62
95-99	1.13	.40	.66	1.11	.66	.87	.66	.56	.87	.56	.56	.87	1.11	.56	.56
96-00	1.29	.72	.52	.81	.52	.81	.56	.22	.81	.72	.54	.81	.54	.54	.72
97-01	1.06	.55	.64	.80	.64	.70	.64	.26	.39	.55	.64	.70	.64	.64	.55
98-02	.94	.73	.54	.73	.54	.67	.73	.54	.54	.73	.54	.67	.67	.54	.73
99-03	.37	.65	.37	1.03	.50	.82	.79	.76	.65	.79	.50	.82	.82	.37	.79

# $(s_4)$ NEX Corr. 7



	AT	BE	DE	DK	ES	FI	FR	UK	GR	IE	IT	LU	NL	PT	SE
AT	1	-.39	.80	-.32	.11	.02	-.89	-.62	.30	-.59	.60	.02	-.26	.84	-.59
BE		1	-.65	-.39	.09	-.39	.15	-.30	-.32	.62	-.61	-.39	-.27	-.48	.62
DE			1	-.07	.44	-.05	-.56	-.35	.06	-.92	.82	-.05	.13	.93	-.92
DK				1	.22	.85	.28	.56	.58	-.14	-.28	.85	.86	-.41	-.14
ES					1	-.03	-.16	-.37	-.18	-.64	.23	-.03	.53	.30	-.64
FI						1	-.13	.30	.86	-.04	-.29	1	.56	-.31	-.04
FR							1	.82	-.29	.47	-.47	-.13	.35	-.67	.47
UK								1	.21	.34	-.40	.30	.50	-.57	.34
GR									1	.05	-.35	.86	.40	-.16	.05
IE										1	-.82	-.04	-.28	-.81	1
IT											1	-.29	-.24	.90	-.82
LU												1	.56	-.31	-.04
NL													1	-.25	-.28
PT														1	-.81
SE															1

# N.B. Randomized $C'$ case

shuffled time

GDP

	AT	BE	DE	DK	ES	FI	FR	UK	GR	IE	IT	LU	NL	PT	SE
AT	1	.19	-.07	-.28	.23	-.23	.45	.55	-.47	.07	-.35	.28	-.43	.29	-.49
BE		1	.51	.10	-.10	-.47	.16	.24	-.35	-.48	-.61	.41	.07	-.55	.18
DE			1	.53	.24	-.22	.70	-.22	-.48	-.50	-.11	-.34	-.02	.24	.16
DK				1	-.32	.19	.19	.27	-.20	-.64	-.22	-.67	-.15	.36	.34
ES					1	.42	.58	-.57	-.60	.32	.66	-.21	.06	.37	.15
FI						1	.00	-.16	-.17	-.02	.71	-.67	.28	.33	.43
FR							1	-.06	-.53	-.33	.17	-.44	.00	.62	-.32
UK								1	.00	-.46	-.68	.09	-.23	.00	-.32
GR									1	-.05	.08	.10	.50	-.37	-.42
IE										1	.26	.44	-.44	.05	.08
IT											1	-.52	.47	.32	.10
LU												1	-.22	-.67	-.12
NL													1	-.40	-.12
PT														1	-.21
SE															1

*conclusion is : (here) no structure*

# Randomized world





# EU25 Summary of Tables

	AT	BE	DE	DK	ES	FI	FR	UK	GR	IE	IT	LU	NL	PT	SE
94-98	.67	.86	.86	.86	.40	.40	.67	.86	.40	.86	.86	.40	.40	.86	.86
95-99	.60	.65	.52	.71	.21	.77	.45	.77	.37	.65	.90	.37	.23	.83	.52
96-00	.58	.32	.46	.61	.34	.81	.46	.32	.32	.53	.32	.20	.60	.60	.46
97-01	.48	.30	.48	.30	.28	.42	.48	.44	.68	.38	.68	.14	.28	.28	.48
98-02	.43	.26	.19	.19	.21	.43	.19	.19	1.04	.29	.44	.12	.21	.21	.29
99-03	.25	.23	.19	.19	.29	.26	.19	.37	1.15	.26	.37	.23	.19	.19	.28
00-04	.27	.27	.17	.26	.28	.27	.21	.27	.53	.50	.28	.27	.21	.21	.27

	AT	BE	DE	DK	ES	FI	FR	UK	GR	IE	IT	LU	NL	PT	SE
94-98	.88	.65	.85	.88	.65	.37	.65	.65	.65	.65	.37	.65	.65	.65	.65
95-99	.79	.79	.79	.81	.79	.41	.79	.79	.93	.79	.53	.59	.79	.79	.79
96-00	1.02	1.02	1.02	1.02	1.02	1.02	1.02	1.02	1.02	1.02	.26	1.02	1.02	1.02	1.02
97-01	.51	.51	.51	.65	.51	.73	.88	.51	.65	.51	.33	.88	.51	.51	.51
98-02	.52	.52	.52	.96	.52	.66	.95	.65	.96	.52	.35	1.19	.52	.52	.52
99-03	.45	.42	.45	1.00	.45	.53	.40	.46	1.00	.42	.30	.92	.45	.45	.45
00-04	.88	.65	.85	.88	.65	.37	.65	.65	.65	.65	.37	.65	.65	.65	.65

GDP | FCE  
GCF | NEX

	AT	BE	DE	DK	ES	FI	FR	UK	GR	IE	IT	LU	NL	PT	SE
94-98	.51	.48	.59	.52	.66	.48	.66	.58	.89	.67	.38	.85	.67	.37	.51
95-99	.47	.46	.75	.49	.54	.46	.54	.61	.75	.49	.33	.83	.49	.39	.58
96-00	.75	.78	.75	.78	.75	.78	.75	.58	.75	.84	.32	.32	.48	.20	.75
97-01	.70	.47	.70	.62	.70	.62	.70	.57	.70	.38	.63	.29	.29	.09	.70
98-02	.46	.46	.46	.68	.46	.68	.46	.61	.46	.46	1.13	.46	.46	.46	.46
99-03	.70	.70	.70	.88	.70	.88	.70	.70	.70	.70	1.07	.70	.70	.70	.70

	AT	BE	DE	DK	ES	FI	FR	UK	GR	IE	IT	LU	NL	PT	SE
94-98	1.27	.19	.65	.89	.45	.80	.65	.62	.75	.62	.62	.80	.64	.62	.62
95-99	1.13	.40	.66	1.11	.66	.87	.66	.56	.87	.56	.56	.87	1.11	.56	.56
96-00	1.29	.72	.52	.81	.52	.81	.56	.22	.81	.72	.54	.81	.54	.54	.72
97-01	1.06	.55	.64	.80	.64	.70	.64	.26	.39	.55	.64	.70	.64	.64	.55
98-02	.94	.73	.54	.73	.54	.67	.73	.54	.54	.73	.54	.67	.67	.54	.73
99-03	.37	.65	.37	1.03	.50	.82	.79	.76	.65	.79	.50	.82	.82	.37	.79

# Summary C Tables

EU25

	AT	BE	DE	DK	ES	FI	FR	UK	GR	IE	IT	LU	NL	PT	SE
AT	1	.77	.88	.88	.33	.69	.88	.69	-.69	.75	.71	.42	.61	.89	.85
BE		1	.88	.90	.41	.27	.80	.94	-.59	.92	.83	.85	.23	.90	.91
DE			1	.90	.61	.35	.98	.86	-.65	.85	.78	.61	.52	.86	.99
DK				1	.50	.58	.87	.84	-.80	.93	.67	.77	.58	.99	.88
ES					1	-.10	.61	.34	-.38	.55	.05	.36	.66	.37	.64
FI						1	.42	.25	-.62	.34	.27	.14	.60	.64	.26
FR							1	.79	-.71	.81	.73	.52	.60	.82	.95
UK								1	-.52	.82	.90	.85	.12	.86	.86
GR									1	-.82	-.38	-.56	-.62	-.76	-.60
IE										1	.63	.85	.43	.89	.87
IT											1	.59	-.05	.73	.77
LU												1	.06	.77	.65
NL													1	.50	.47
PT														1	.84
SE															1

	AT	BE	DE	DK	ES	FI	FR	UK	GR	IE	IT	LU	NL	PT	SE
AT	1	.92	1	.23	.92	.21	.38	.87	.03	.92	.07	-.34	.92	.92	.92
BE		1	.94	.23	1	.45	.56	.97	.28	1	.06	-.15	1	1	1
DE			1	.24	.93	.24	.40	.89	.07	.94	.07	-.32	.93	.93	.93
DK				1	.26	.22	-.14	.35	.75	.23	-.41	.44	.26	.26	.26
ES					1	.45	.53	.97	.31	1	.04	-.15	1	1	1
FI						1	.65	.49	.34	.45	-.68	.68	.45	.45	.45
FR							1	.64	.05	.56	-.05	.38	.53	.53	.53
UK								1	.40	.97	.03	.02	.97	.97	.97
GR									1	.28	-.11	.45	.31	.31	.31
IE										1	.06	-.15	1	1	1
IT											1	-.68	.04	.04	.04
LU												1	-.15	-.15	-.15
NL													1	1	1
PT														1	1
SE															1

GDP | FCE  
GCF | NEX

	AT	BE	DE	DK	ES	FI	FR	UK	GR	IE	IT	LU	NL	PT	SE
AT	1	.76	.59	.68	.88	.69	.88	.10	.19	.45	-.04	-.58	-.12	-.26	.94
BE		1	.47	.81	.67	.79	.67	.35	.15	.85	-.02	-.27	.32	.15	.73
DE			1	.10	.64	.09	.64	.05	.55	.30	-.57	-.02	-.08	-.25	.81
DK				1	.41	1	.41	.61	-.32	.50	.56	-.40	.24	.39	.55
ES					1	.40	1	-.04	.61	.58	-.35	-.26	.11	-.29	.83
FI						1	.40	.58	-.37	.46	.57	-.46	.17	.35	.56
FR							1	-.04	.61	.58	-.35	-.26	.11	-.29	.83
UK								1	-.21	.20	.63	.37	.61	.91	.12
GR									1	.44	-.76	.45	.37	-.20	.27
IE										1	-.26	.10	.62	.21	.40
IT											1	-.15	.12	.60	-.21
LU												1	.73	.60	-.46
NL													1	.78	-.17
PT														1	-.27
SE															1

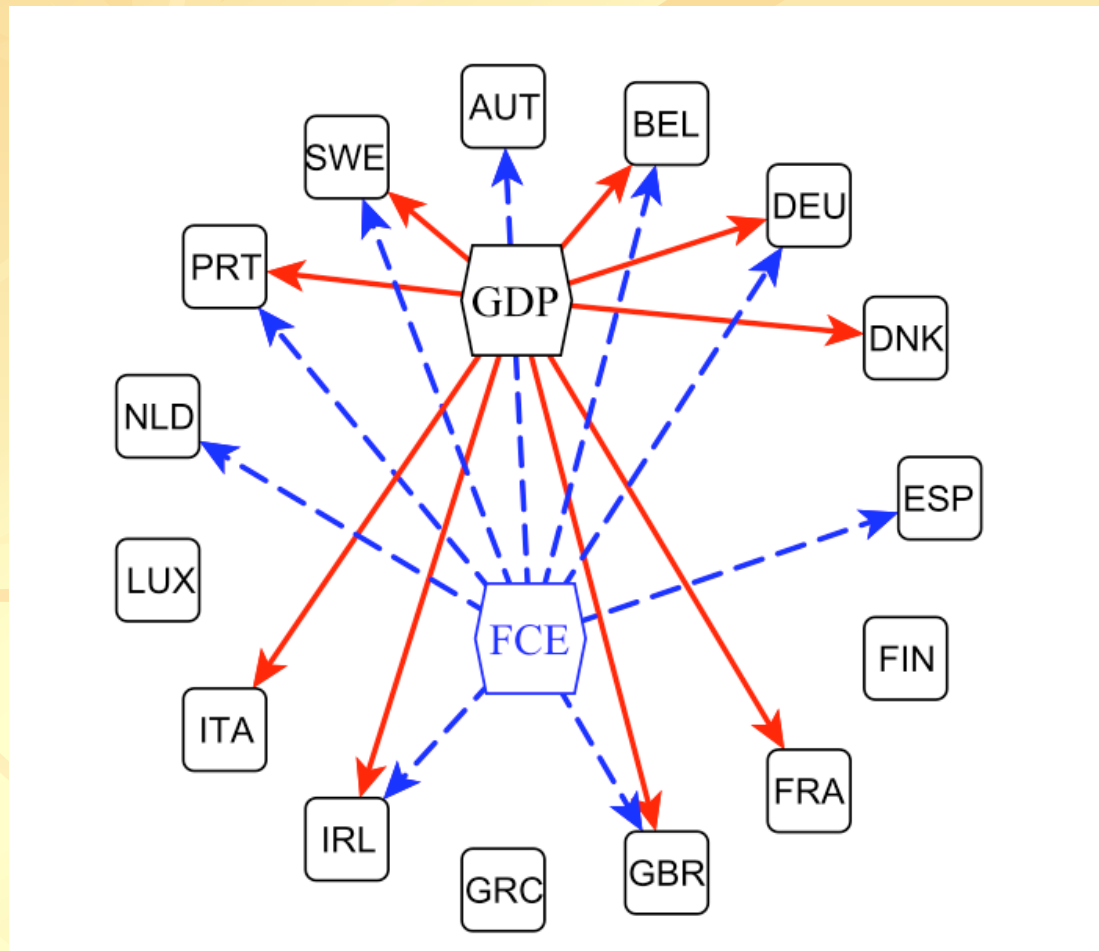
	AT	BE	DE	DK	ES	FI	FR	UK	GR	IE	IT	LU	NL	PT	SE
AT	1	-.39	.80	-.32	.11	.02	-.89	-.62	.30	-.59	.60	.02	-.26	.84	-.59
BE		1	-.65	-.39	.09	-.39	.15	-.30	-.32	.62	-.61	-.39	-.27	-.48	.62
DE			1	-.07	.44	-.05	-.56	-.35	.06	-.92	.82	-.05	.13	.93	-.92
DK				1	.22	.85	.28	.56	.58	-.14	-.28	.85	.86	-.41	-.14
ES					1	-.03	-.16	-.37	-.18	-.64	.23	-.03	.53	.30	-.64
FI						1	-.13	.30	.86	-.04	-.29	1	.56	-.31	-.04
FR							1	.82	-.29	.47	-.47	-.13	.35	-.67	.47
UK								1	.21	.34	-.40	.30	.50	-.57	.34
GR									1	.05	-.35	.86	.40	-.16	.05
IE										1	-.82	-.04	-.28	-.81	1
IT											1	-.29	-.24	.90	-.82
LU												1	.56	-.31	-.04
NL													1	-.25	-.28
PT														1	-.81
SE															1

## 6. Hamiltonian structure (1)

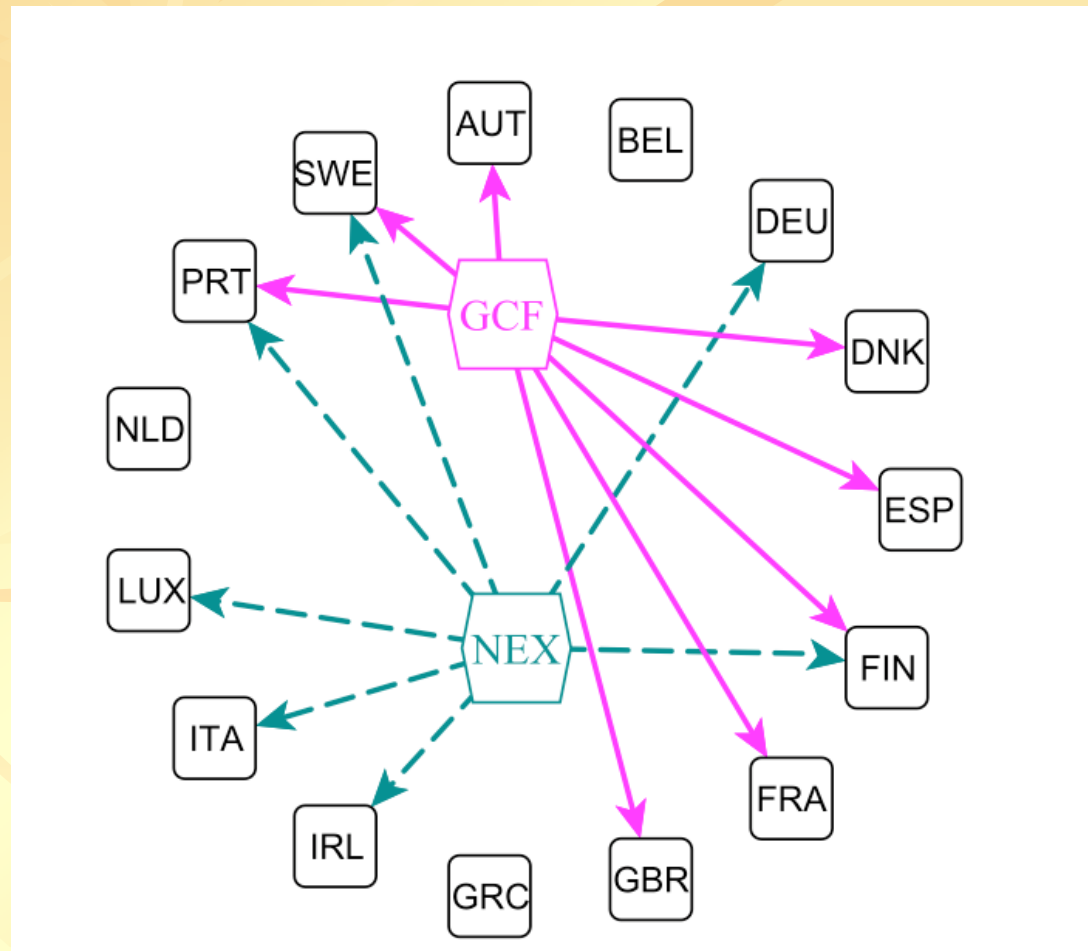
**Table 5** The EU-15 country clustering. The second column displays the eigenvector whose components are used for building the classification scheme. The groups into parentheses are the second-order clusters

INDICATOR	EVC	Clusters
Gross Domestic Product (GDP)	$v_2$	BEL-GBR-ITA-LUX AUT-DEU-DNK-FRA-PRT (ESP-FIN-NLD)
Final Consumption Expenditure	$v_2$	AUT-DEU (DNK-FIN-FRA-GRC-LUX)
Gross Capital Formation	$v_2$	BEL-DNK-FIN-GBR-PRT ESP-FRA
Net Exports	$v_1$	AUT-DEU-ITA-PRT DNK-FRA-GBR-IRL-SWE
Consumer Price Index	$v_1$	DEU-ITA-GRC-LUX FIN-FRA-IRL
Rate of Interest	$v_2$	GBR-LUX-SWE All the others, except for GRC
Labour Force	$v_1$	AUT-BEL-ESP-GBR-LUX DEU-DNK-ITA-PRT
Unemployment	$v_1$	AUT-DEU-FRA-GRC-ITA-SWE DNK-ESP-FIN-GBR-IRL-LUX-NLD
GDP/hour worked	$v_1$	DEU-FRA-LUX-PRT (ESP-GRC-SWE)
GDP/capita	$v_2$	BEL-DEU-FRA-GRC-ITA-LUX-SWE ESP-FIN-IRL-NLD-PRT
Gini Coefficient	$v_1$	AUT-BEL-DEU-DNK-GBR-LUX-NLD-SWE ESP-FRA-GRC-IRL-ITA-PRT

# GDP & FCE

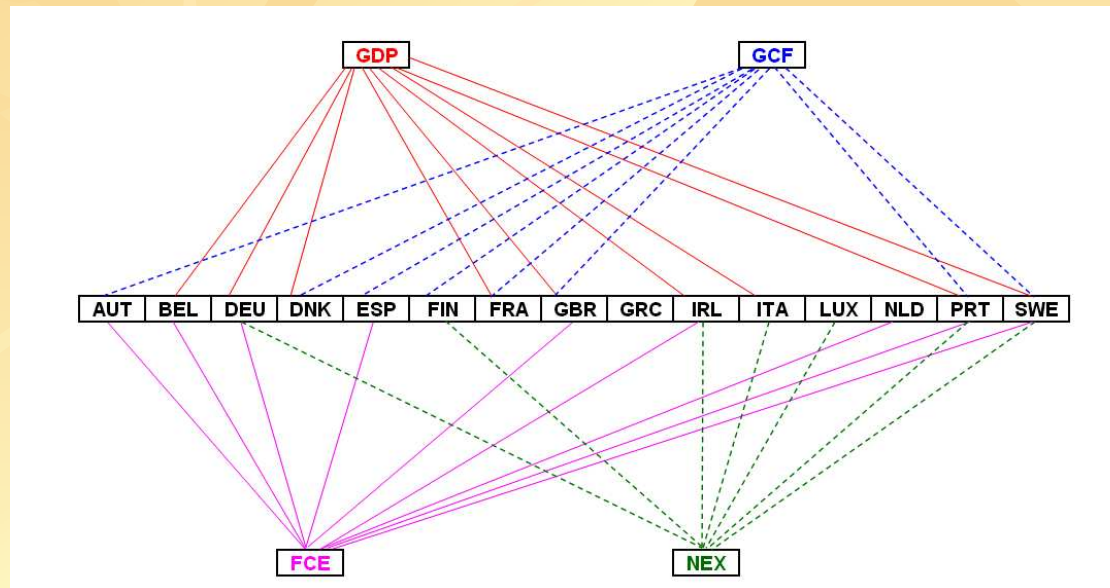


# GCF & NEX



# Hamiltonian structure (2)

$C^2 > 0.81$



- $H = (\text{LUX})(s_4) + (\text{NLD})(s_2)$   
 $+ (\text{AUT})(s_2, s_3) + (\text{BEL})(s_1, s_2) + (\text{DNK})(s_1, s_3)$   
 $+ (\text{ESP})(s_2, s_3) + (\text{FIN})(s_3, s_4) + (\text{FRA})(s_1, s_3) + (\text{ITA})(s_1, s_4)$   
 $+ (\text{GBR})(s_1, s_2, s_3) + (\text{DEU})(s_1, s_2, s_4) + (\text{IRL})(s_1, s_2, s_4)$   
 $+ (\text{PRT})(s_1, s_2, s_3, s_4) + (\text{SWE})(s_1, s_2, s_3, s_4).$



# Hamiltonian structure (3)

$$\begin{aligned} H = & \textit{AUT}(x_1, x_2, x_3, x_4, y_2, z_1, z_2, z_3, w_1, w_2) + \textit{BEL}(x_1, x_2, x_3, y_1, y_2, z_1, z_3, w_1, w_2) + \\ & \textit{DEU}(x_1, x_2, x_4, y_1, y_2, z_1, z_2, z_3, w_1, w_2) + \textit{DNK}(x_1, x_3, x_4, y_2, z_1, z_2, w_1, w_2) + \\ & \textit{ESP}(x_2, x_3, y_2, z_1, z_2, w_1, w_2) + \textit{FIN}(x_3, x_4, y_1, y_2, z_2, z_3, w_1, w_2) + \\ & \textit{FRA}(x_1, x_3, x_4, y_1, y_2, z_2, z_3, w_1, w_2) + \textit{GBR}(x_1, x_2, x_3, x_4, y_1, y_2, z_1, z_2, z_3, w_1, w_2) + \\ & \textit{GRC}(x_4, y_1, z_2, w_1, w_2) + \textit{IRL}(x_1, x_2, x_3, x_4, y_1, y_2, z_2, w_1, w_2) + \\ & \textit{ITA}(x_1, x_4, y_1, y_2, z_1, z_2, w_1, w_2) + \textit{LUX}(x_1, x_4, y_1, y_2, z_1, z_2, z_3, w_1, w_2) + \\ & \textit{NLD}(x_2, x_4, y_2, z_2, w_1, w_2) + \textit{PRT}(x_1, x_2, x_3, x_4, y_1, y_2, z_1, z_2, z_3, w_1, w_2) + \\ & \textit{SWE}(x_1, x_2, x_3, x_4, y_1, y_2, z_2, w_1, w_2) \end{aligned}$$



# “thermodynamics” results

- *Probability distribution :*
- *Entropy :*

$$p_{\alpha}(s_{\alpha}) = \sum_{s \in s_{\alpha}} p(s)$$

$$S_{\alpha}(s_{\alpha}) = - \sum_{s_{\alpha}} p_{\alpha}(s_{\alpha}) \ln p_{\alpha}(s_{\alpha})$$

Function Nodes	Cluster	Number of links	Number of possible links	Probability	Entropy
GDP-FCE-GCF	AUT-BEL-DNK-ESP-FRA-GBR-NLD	14	28	0.500	0.347
FCE-GCF-NEX	AUT-ESP-FIN-LUX-NLD	8	20	0.400	0.367
GDP-FCE-NEX	BEL-DEU-IRL-ITA-LUX-NLD	12	24	0.500	0.347
GDP-GCF-NEX	DNK-FIN-FRA-ITA-LUX	9	20	0.450	0.359

# “Partial Conclusion”

- Network construction
  - Distance filtering method
    - “Correlation measure”
- Statistical physics
  - Entropy criterion for (evolving) clusters
    - Maximum : no GDP
    - Minimum : necessarily GDP and FCE
  - Relations between macro-economic indicators

# Network characteristics (5)

## *Sensitivity degree*

**EU25**  
**(94-03)**

$$(\chi_i)_\alpha = \sum_{\substack{i,j=1 \\ i \neq j}}^N (\hat{C}_{ij})^2$$

$(T=5)$

### ■ “Hierarchy”

- *Gross Domestic Product (GDP)*
- *Final Consumption Expenditure (FCE)*
- *Gross Capital Formation (GCF)*
- *Net Exports (NEX)*

GDP		FCE		GCF		NEX	
DK	9.08	BE	8.34	AT	4.99	PT	5.23
PT	8.71	IE	8.34	SE	4.69	DE	4.92
DE	8.68	ES	8.32	ES	4.66	IE	4.76
SE	8.47	NL	8.32	FR	4.66	SE	4.76
IE	8.26	PT	8.32	BE	4.58	IT	4.41
BE	8.25	SE	8.32	DK	4.18	AT	3.99
FR	8.21	UK	8.14	FI	4.09	DK	3.50
AT	7.60	DE	7.42	IE	3.04	FR	3.24
UK	7.59	AT	7.15	PT	2.89	FI	3.23
IT	5.68	FR	3.07	DE	2.85	LU	3.23
GR	5.64	FI	3.06	IT	2.70	UK	2.91
LU	5.40	LU	1.81	UK	2.68	BE	2.71
NL	3.25	DK	1.61	GR	2.63	NL	2.63
ES	2.97	GR	1.60	LU	2.39	GR	2.49
FI	2.68	IT	1.13	NL	2.31	ES	1.69

# 7. Conclusions

- Beyond bipartite graphs : (very) complex networks
- (New?) “hamiltonian” / “thermodynamic” method
  - *To search for Clusters*
  - *To present a Hierarchy*
- Economic conclusions/interpretations
  - *Flexibility*
  - *Movement correlations*
  - **Globalization**
- Models
  - *Control & Forecasting*

*This should allow to discuss leaders  
& followers, loosely or highly  
connected countries.*

and  
prepare some modeling with dynamics:  
*country = heterogeneous agent*  
characterized by  
Spins or Bits or Pins or Tags

...

*Let me*

- *thank Dr. Peter Denteneer for the invitation*
- *thank you ALL for your attention ...*

*any comment ?*

# Doel #?

*PCC : For ONE MEI ... Network*

*CPCC : For 4 (.. 11) MEI ... Networks*

*AOI*

*Double threshold attrition*

*Distance evolutions*

*PCC*

*CPCC*

*...*



# Main Points summary/review:

- *Data : GDP as a macroeconomy index*
- *Analysis on **increment** (fluctuation) correlations is performed for different time windows (does it exist an optimal one ?)*
- *Mathematical - statistical approach (“distances”)*
- *=> **Weighted** networks  $w_{ij}(t,T)$*
- *Results displayed as a function of time through different clustering techniques*
  - *Adjacency matrix (eigenvalues, eigenvectors)*
  - *Overlapping/**hierarchy** coefficient*
  - *Percolation approach (with **level-criterion**)*
  - ***Two** possible attrition processes*
- *Networks/clusters of countries are observed*

# N.B.

- *The system is represented by a **network**, nodes being the countries; links are « **weights** » (or GDP fluctuations).*
- *In order to extract structures from the network, we **average** the time correlations in different windows.*
  - *=> « **average degree** », etc ...*
- *The matrix-based method reveals the emergence of a number of « **common factors** », through the main eigenvectors (Kaiser criterion and Cattell scree test).*
- *This leads to a Hamiltonian-like formulation and can be developped into statistical thermodynamics ideas*

## N.B. btw

- *Such a measure of « **collective habits** » does fit the usual expectations defined by politicians or economists,*
  - « **common factors** ».
- *It reveals geographical and political connexions.*
- *It reveals ... « **statistical** distances ».*
- *We have introduced the*  
*country «**overlapping hierarchy coefficient** ».*

# Ex.

**FRANCE, GDP by Sector of Origin in Current Prices**  
Millions Francs

	ID	1815	1816	1817	1818	1819	1820	1821
AGRICULTURE, HUNTING, FORESTRY AND FISHING	A; B							
AGRICULTURE, HUNTING AND FORESTRY	A							
FISHING	B							
TOTAL INDUSTRY	C; D; E; F	2 710	3 007	3 097	3 412	3 110	3 252	3 379
MINING AND QUARRYING	C							
MANUFACTURING	D							
ELECTRICITY, GAS AND WATER SUPPLY	E							
CONSTRUCTION	F							
WHOLESALE AND RETAIL TRADE; HOTELS AND RES	G; H							
TRADE	G	509	644	792	708	650	662	688
HOTELS AND RESTAURANTS	H							
TRANSPORT, STORAGE AND COMMUNICATIONS	I							
FINANCIAL INTERMEDIATION; REAL ESTATE, RENTII	J; K							
FINANCIAL INTERMEDIATION	J							
REAL ESTATE, RENTING AND BUSINESS ACTIVITIE	K							
COMMUNITY, SOCIAL AND PERSONAL SERVICES	P; MISCs							
PUBLIC ADMINISTRATION AND DEFENCE; COMPU	L							
EDUCATION	M							
HEALTH AND SOCIAL WORK	N							
OTHER COMMUNITY, SOCIAL AND PERSONAL SEF	O	82	103	126	112	102	103	107
PRIVATE HOUSEHOLDS WITH EMPLOYED PERSONS	P	177	182	186	191	195	200	205
EXTRA-TERRITORIAL ORGANIZATION AND BODIES	Q							
TOTAL VALUE ADDED	TVA							
NET INDIRECT TAXES	ITAX							
<b>TOTAL GDP</b>	<b>GDP</b>	<b>7 378</b>	<b>8 895</b>	<b>10 594</b>	<b>9 554</b>	<b>8 866</b>	<b>8 966</b>	<b>9 268</b>

	ID	1815	1816	1817	1818	1819	1820	1821
AGRICULTURE, HUNTING, FORESTRY AND FISHING	A; B							
AGRICULTURE, HUNTING AND FORESTRY	A							
Agriculture	1	2 752	3 839	5 268	4 053	3 715	3 637	3 764
Forestry	2							
FISHING	B							

*Should we expect a “grand theory” ?*

*Consider Tocqueville’s “warning”...*

I hate...these absolute systems which make all the events in history depend on primary causes, linking one to another by an inevitable chain, and which, so to speak, take out people from the general history of mankind. I find them narrow in their pretended grandeur, and false under their guise of mathematical truths. I believe, whatever the view of the writers who have invented these sublime theories to nourish their own vanity and to facilitate their work, that many of the important historical facts cannot be explained but by the accidental circumstances, and that many others remain inexplicable. And that finally, chance, or rather that mixing of the secondary causes, which we thus call, since we do not know how to tell them apart, explain a lot of what we see on the world stage. But I strongly believe that chance does not do anything which is not prepared in advance. The existing reality, nature of the institutions, state of mind of people, customs, are the raw materials with which chance constructs the facts which surprise and awe us.

# App. 1 Theil index

Theil index:

$$Th_A(t, T) = \frac{1}{T} \sum_{i=t}^{t+T} \left( \frac{A(i)}{\langle A \rangle_{(t,T)}} \ln \frac{A(i)}{\langle A \rangle_{(t,T)}} \right)$$

Manhattan distance:

$$d_{le}(A, B)_{(t,T_1,T_2)} = |\langle Th_A(t, T_1) - Th_B(t, T_1) \rangle_{(t,T_2)}|$$

# ***Henri Theil***



- **13 October 1924 in Amsterdam, died 2000**
- **a Dutch econometrician**
- **graduated from the University of Amsterdam**
- **succeeded to Jan Tinbergen at the Erasmus University Rotterdam**
- **later in Chicago and at the University of Florida.**



## *Gini and Theil Index*

- In **Gini** terms:

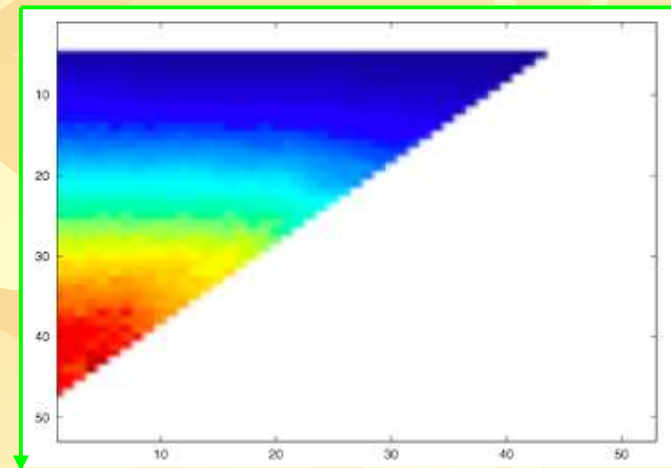
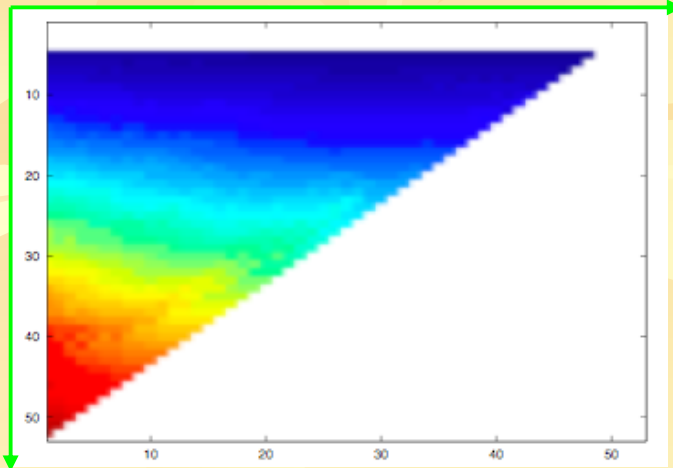
$$\sum_{i=1}^n G_i p_i \pi_i + \frac{1}{\mu} \sum_i^n \sum_{j>i}^n (y_j - y_i) p_i p_j + L$$

where  $G_i$  = individual country Gini coeff.,  $\pi_i$  = income share,  $y_i$  = country income,  $p_i$  = popul. share,  
 $\mu$  = overall mean income,  $n$  = number of countries

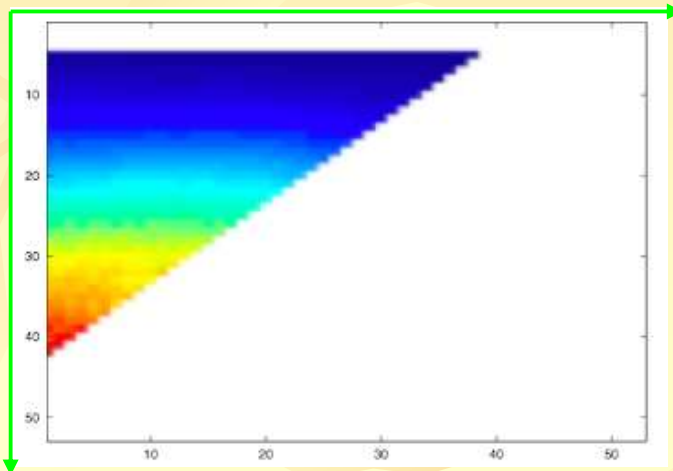
- In **Theil**:

$$\sum_{i=1}^n p_i T_i + \sum_{i=1}^n p_i \ln \frac{\mu}{y_i}$$

# 1 UMLP



«  $x$  » :  $T_2$  *correl.*



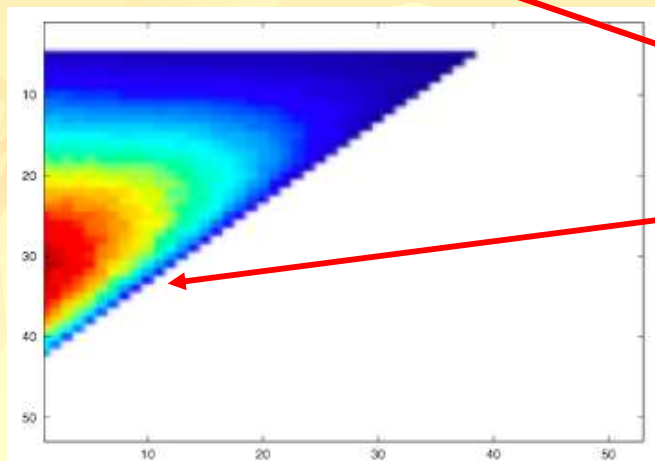
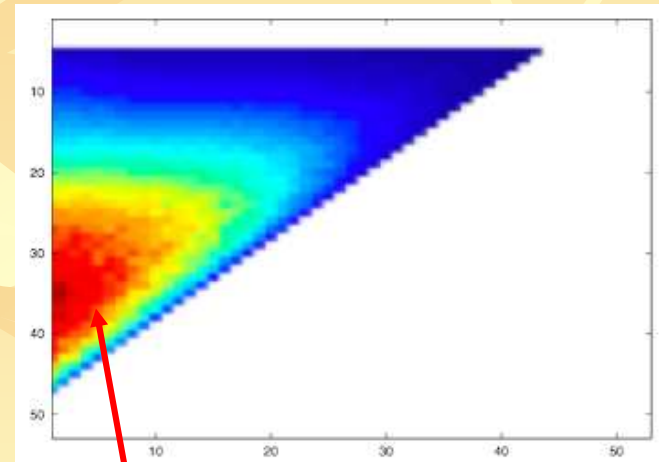
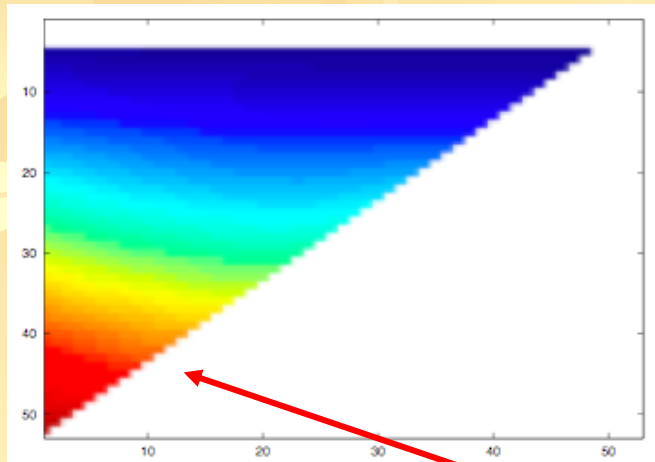
«  $y$  » :  $T_1$

*mapping*

$$\tau$$

0	5
10	

## 2 BMLP

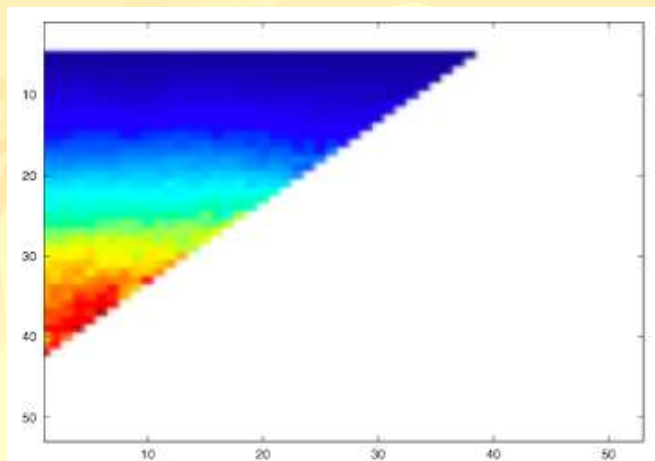
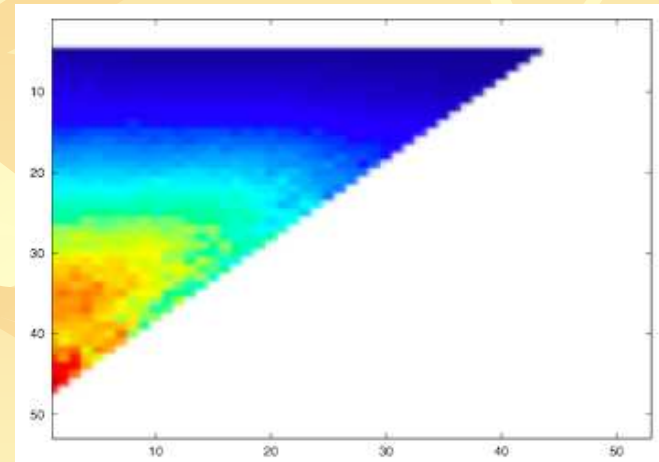
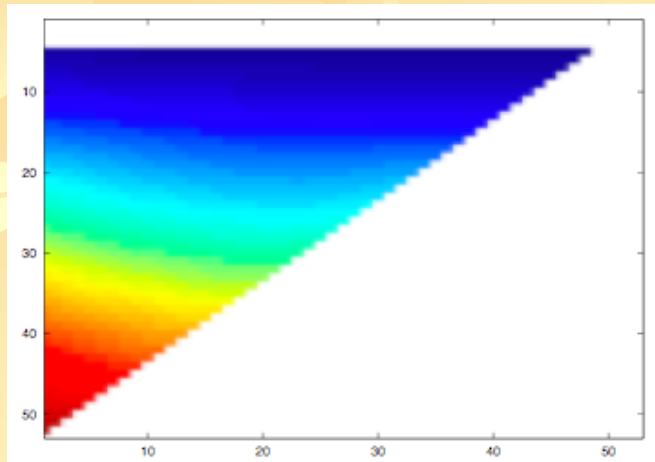


*« hot » (& « cold ») regions :*

*« hot » : large  $T_1$ , small  $T_2$*

*« cold » : small  $T_1$ , large  $T_2$*

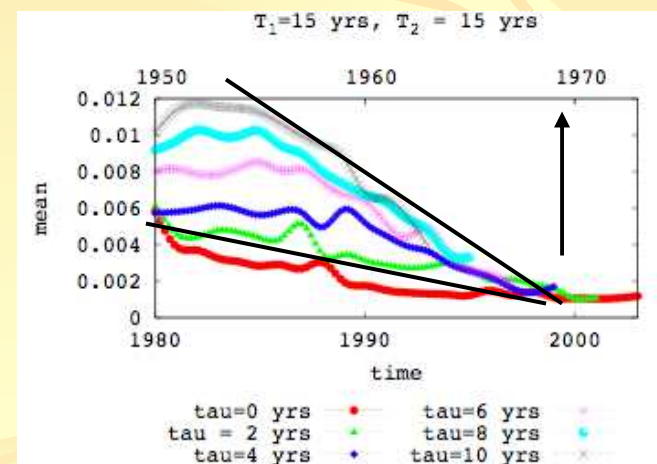
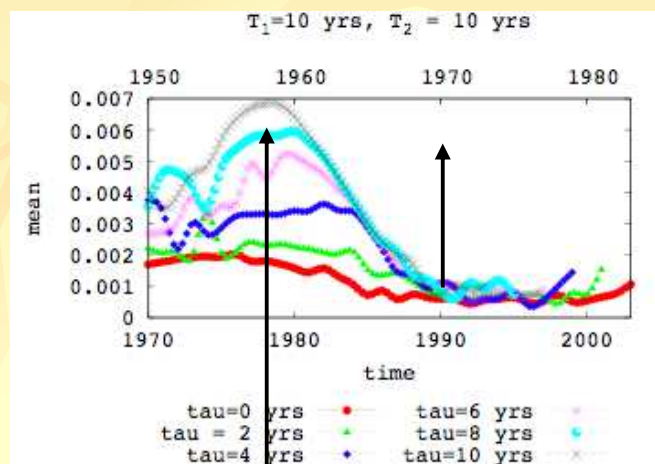
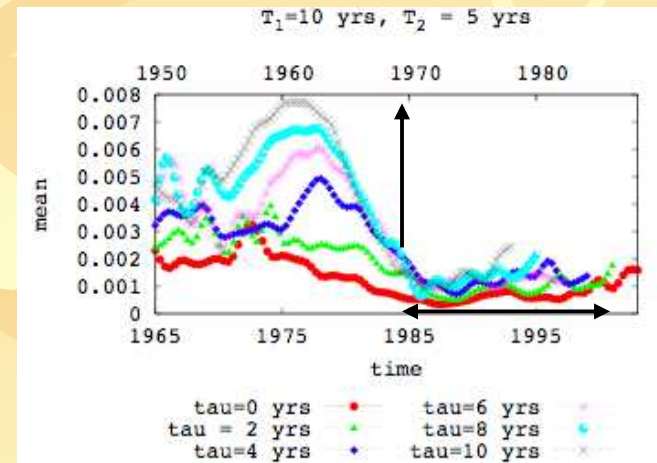
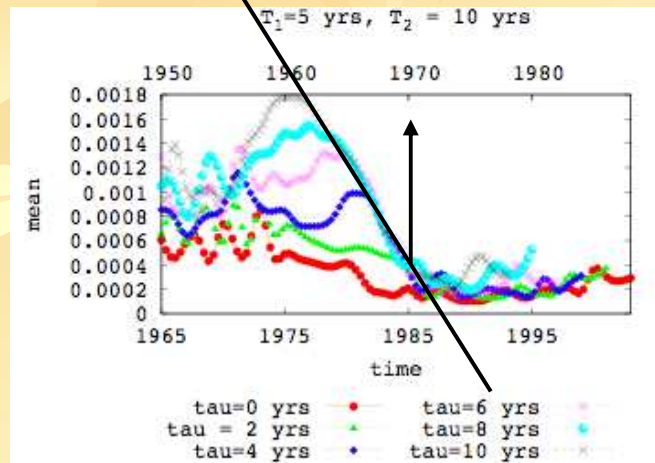
# 3 LMST



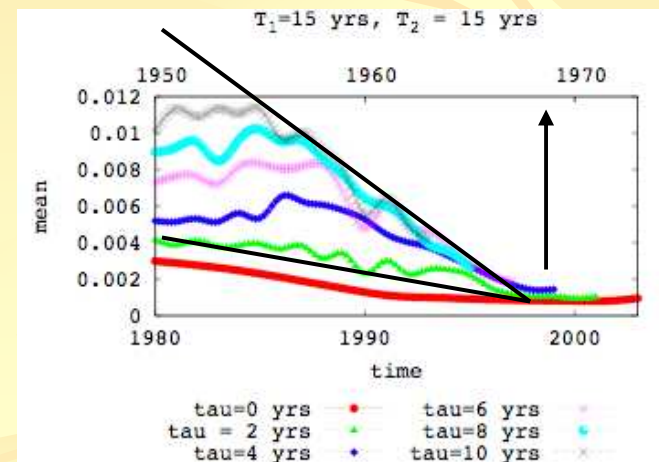
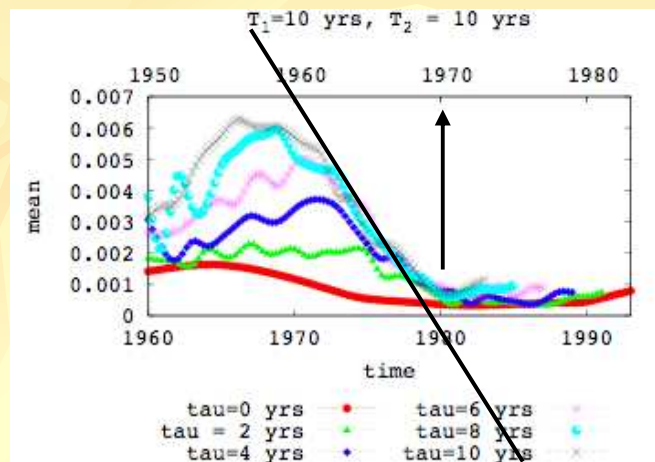
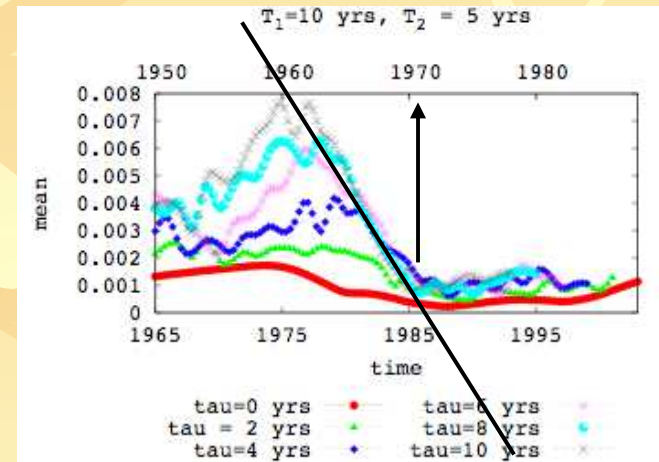
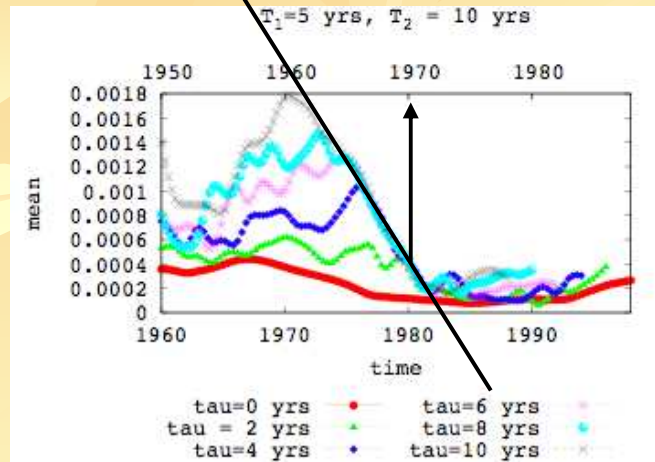
« *hot* » : large  $T_1$  & small  $T_2$

« *cold* » : large  $T_2$  & small  $T_1$

# 7,8,9,10 : UMLP

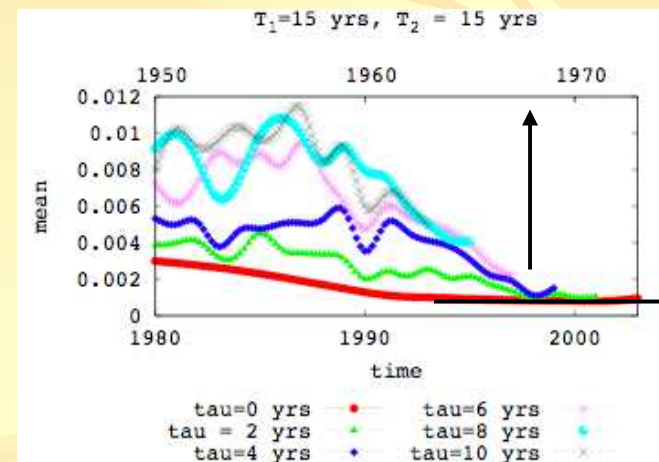
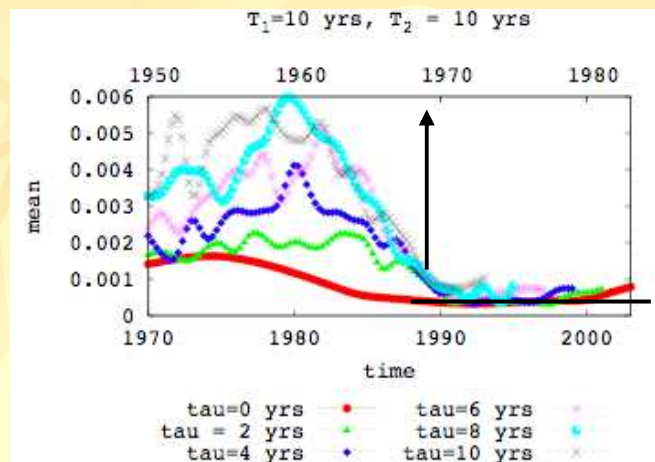
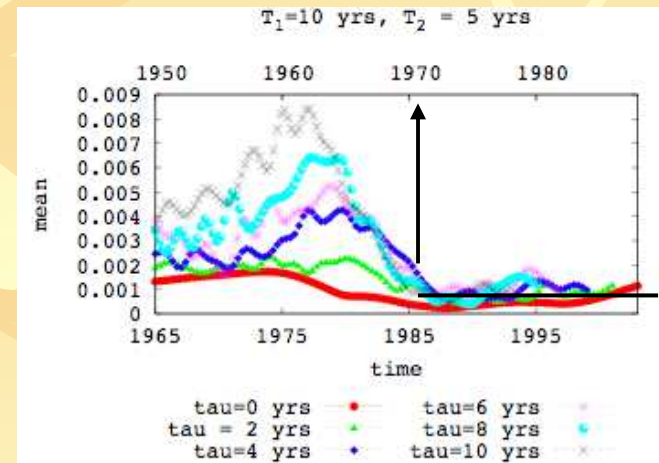
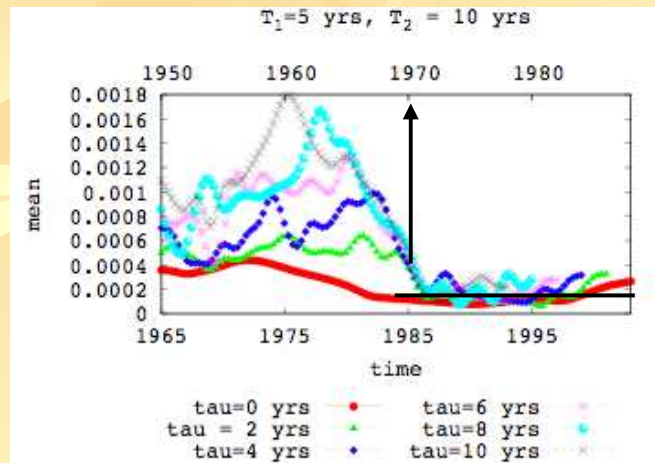


# 11,12,13,14 : BMLP



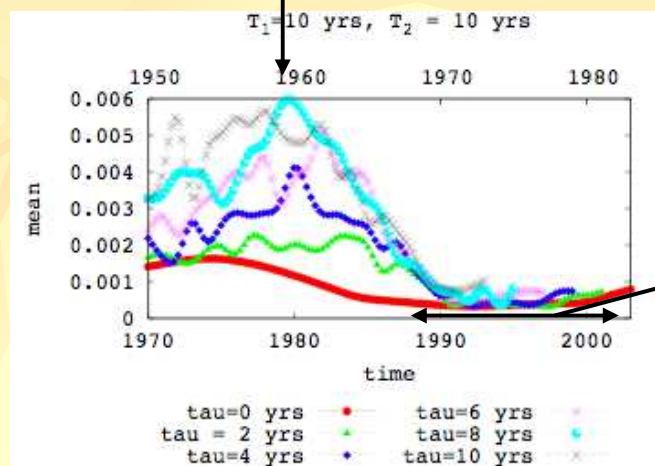
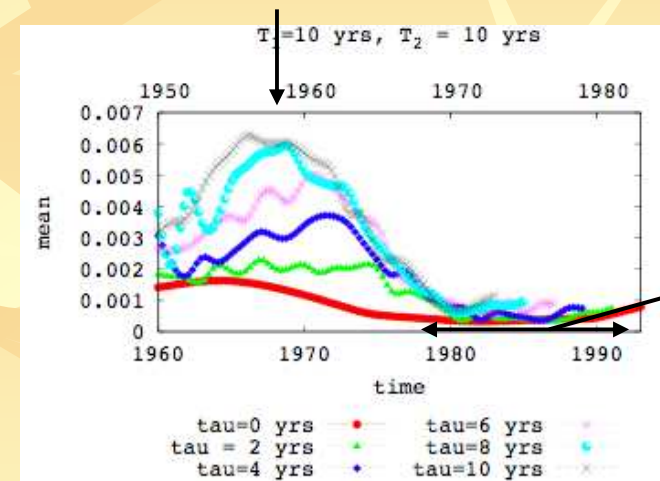
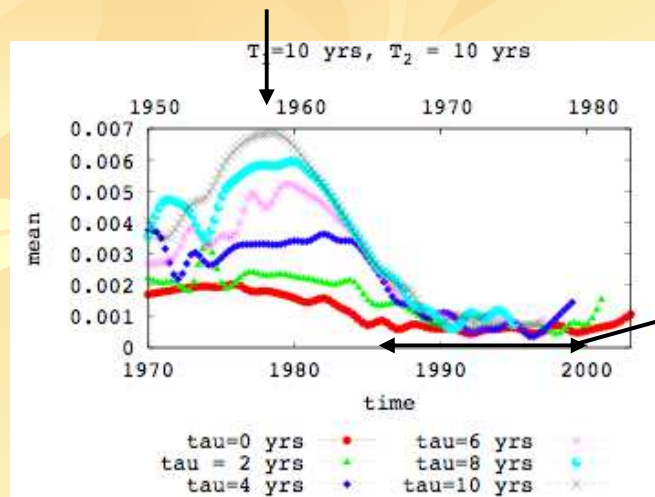


# 15,16,17,18 : LMST





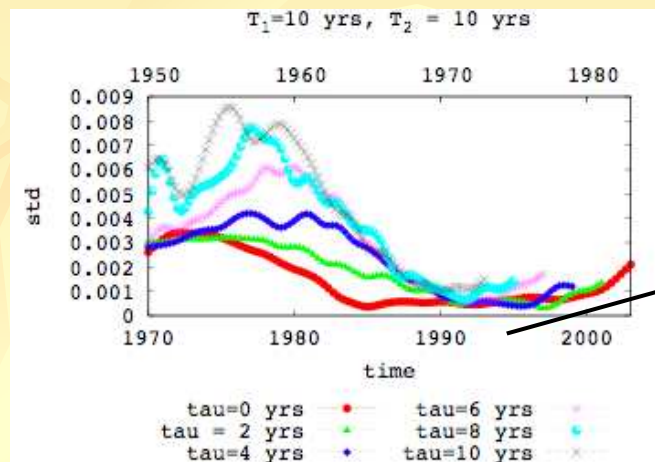
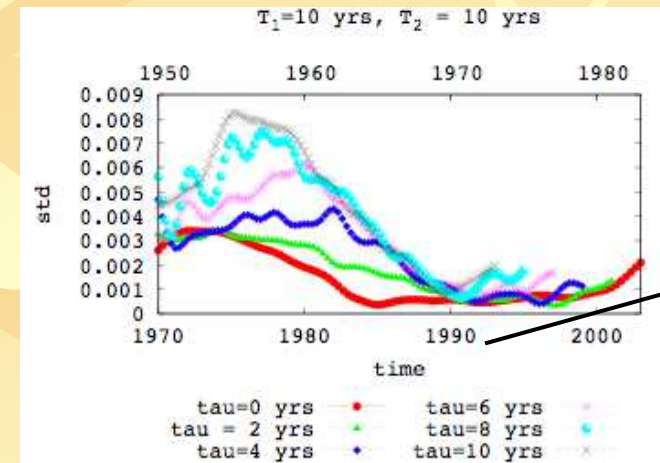
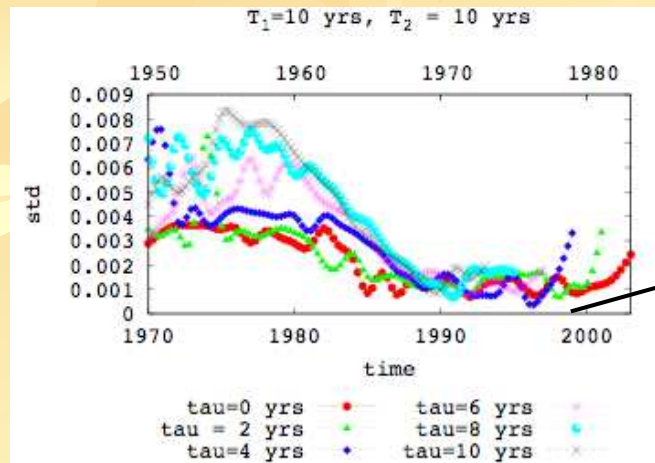
# 9,13,17 : mean (10,10) U,B,L



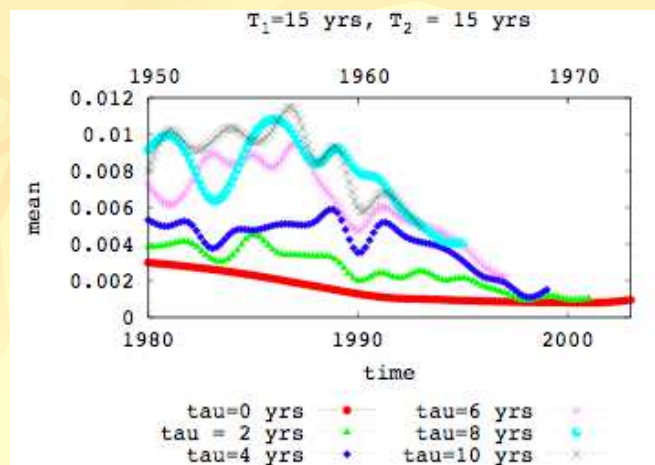
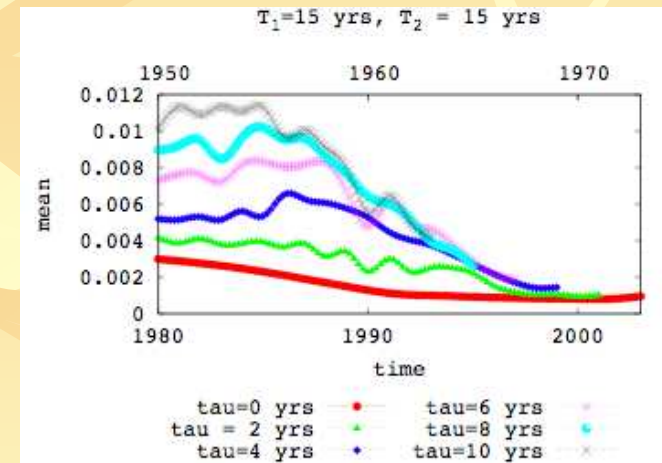
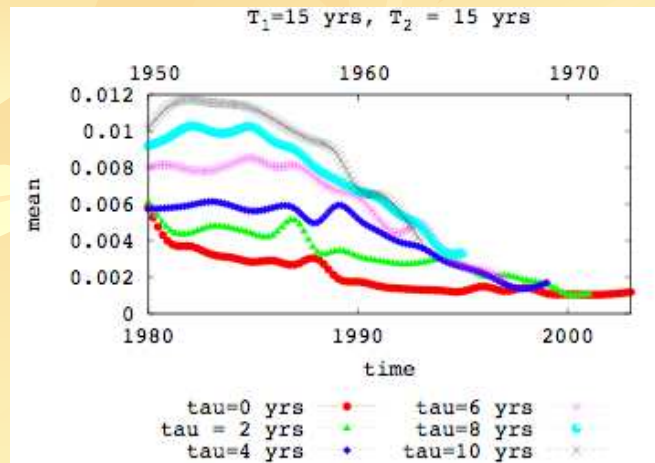
?

$\tau$  effect

# 9,13,17 : std (10,10) U,B,L



# 10,14,18 : mean (15,15) U,B,L



*smooth or rough*

?

# Summary 1-3 : mean U,B,L

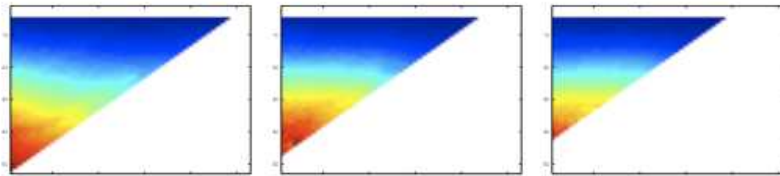


FIG. 1: Mean distance between countries in the case of the Theil distance and UMLP network. The distance is averaged over the network links and the time. The size of  $T_1$  and  $T_2$  are presented on the vertical and horizontal axis respectively. From left to right the three figures correspond to a different time lag value:  $= 0yrs$ ,  $= 5yrs$  and  $= 10yrs$ .

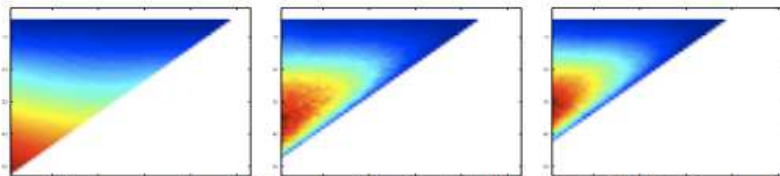


FIG. 2: Mean distance between countries in the case of the Theil distance and BMLP network. The distance is averaged over the network links and the time. The size of  $T_1$  and  $T_2$  are presented on the vertical and horizontal axis respectively. From left to right the three figures correspond to a different time lag value:  $= 0yrs$ ,  $= 5yrs$  and  $= 10yrs$ .

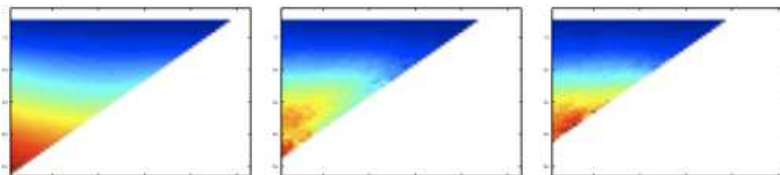


FIG. 3: Mean distance between countries in the case of the Theil distance and LMST network. The distance is averaged over the network links and the time. The size of  $T_1$  and  $T_2$  are presented on the vertical and horizontal axis respectively. From left to right the three figures correspond to a different time lag value:  $= 0yrs$ ,  $= 5yrs$  and  $= 10yrs$ .



# Distance statistics

*from data to networks*

$q = 1$		mean									std								
		max	$T_1$	$T_2$	n-n	min	$T_1$	$T_2$	n-n	max	$T_1$	$T_2$	n-n	min	$T_1$	$T_2$	n-n		
UMLP	= 0y	0.0113	52	1	1	$2 \cdot 10^{-4}$	5	48	1	0.0164	46	1	7	$3.1 \cdot 10^{-4}$	5	48	1		
	= 5y	0.0255	44	4	1	$3.3 \cdot 10^{-4}$	5	43	1	0.0313	44	4	1	$4.4 \cdot 10^{-4}$	5	43	1		
	= 10y	0.0459	42	1	1	$6.6 \cdot 10^{-4}$	5	38	1	0.0466	41	2	1	$8.0 \cdot 10^{-4}$	5	37	2		
BMLP	= 0y	0.0082	52	1	1	$1.6 \cdot 10^{-4}$	5	47	2	0.0082	36	1	17	$2.6 \cdot 10^{-4}$	5	48	1		
	= 5y	0.0136	35	2	12	$5.1 \cdot 10^{-5}$	5	43	1	0.0205	43	1	5	$2.0 \cdot 10^{-4}$	5	43	1		
	= 10y	0.0156	32	1	11	$5.6 \cdot 10^{-5}$	5	38	1	0.0286	34	2	8	$2.9 \cdot 10^{-4}$	5	38	1		
LMST	= 0y	0.0082	52	1	1	$1.6 \cdot 10^{-4}$	5	47	2	0.0082	36	1	17	$2.6 \cdot 10^{-4}$	5	48	1		
	= 5y	0.0254	46	1	2	$3.0 \cdot 10^{-4}$	5	38	6	0.0266	45	3	1	$4.6 \cdot 10^{-4}$	5	38	6		
	= 10y	0.0449	37	6	1	$5.6 \cdot 10^{-4}$	5	38	1	0.0452	42	1	1	$7.4 \cdot 10^{-4}$	5	38	1		

**n-n : number of networks**

# Networks (3)

- *3 types*
  - *Link number :  $M = 20 + 1$*
- *in the analysis both time windows are used simultaneously*
  - *Theil mapping,  $T_1$  and Correlation,  $T_2$*
- *the « official » size of the time window in which data is analyzed is equal to the sum of the Theil mapping and Correlation time windows*
- *... the number of generated networks is equal to the time series length (54) minus the total time window size ( $T_1 + T_2 + \tau$ )*
  - *(... multiplied by the number of time lags  $\tau$ !)*

# “Partial Conclusion”

## *(Theil index) Distance correlations*

- *Mean distance between countries (and std) : the largest for UMLP network and the smallest for LMST network*
- *For large time lag : the mean distance increases*
- *The position where (as a function of Theil mapping and correlation window size) the maximum occurs shifts*
- *With increased time lag: the Theil mapping and correlation window size at which the maximum mean is found is decreasing*
- *This maximum mean distance is observed for large Theil time window size and short correlation time windows*
- *The smallest value of the mean distance is found for the opposite combination of the window parameters*

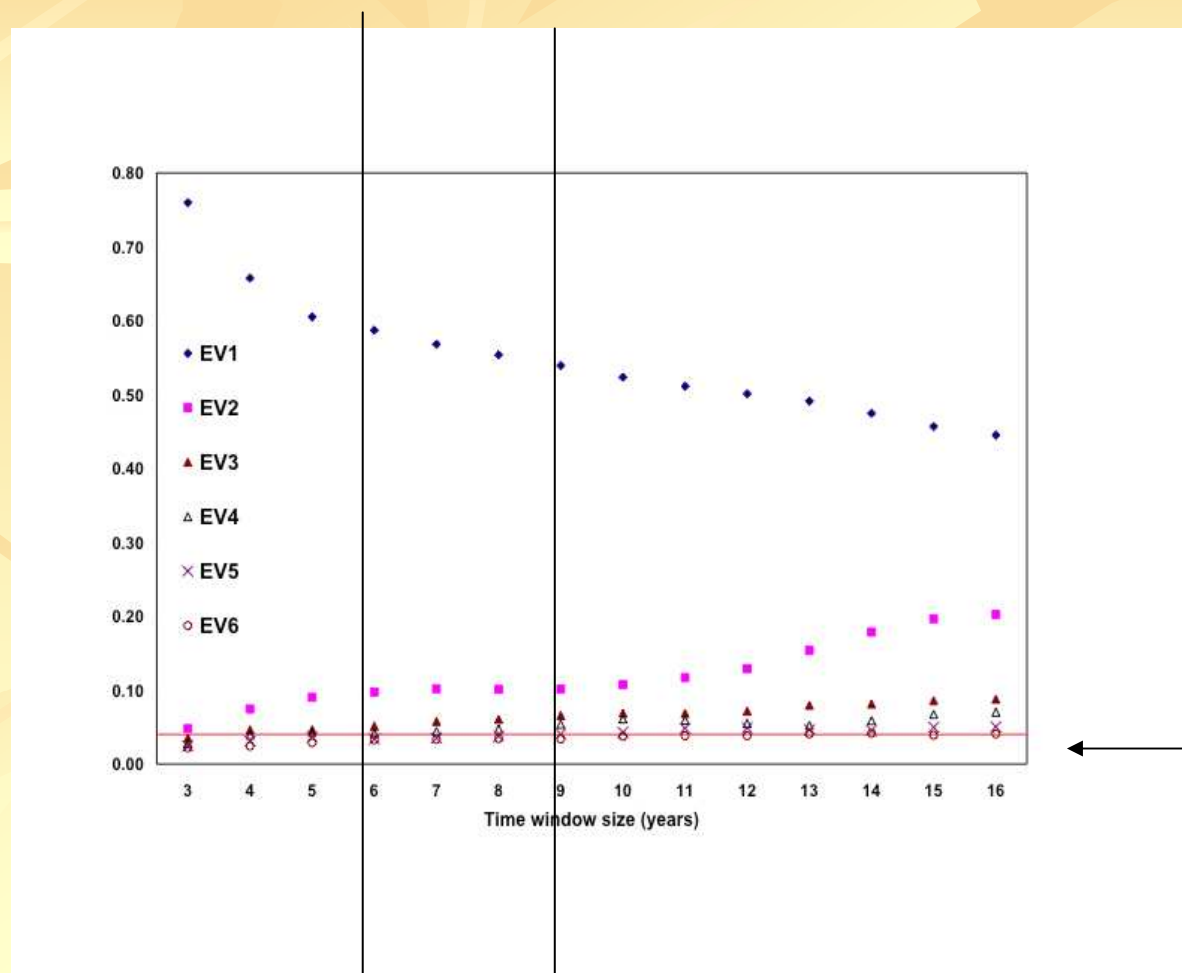


## App. 2 PCA

- *See decrease of EV1*
- *and increase of other EVs.*
- *Related to change in shape of CDF.*
- *Suggests change in number of “**common factors**” (\*) in dynamics of GDP.*

*(\*) : economic, social, political, ...*

# Top six eigenvalues



EU25

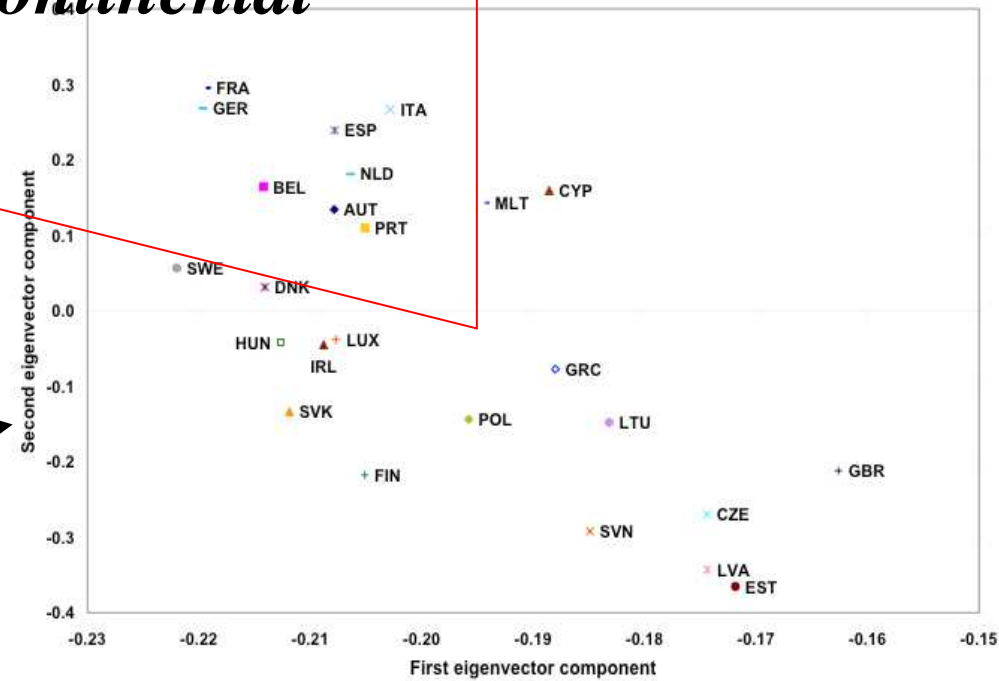
*like  
fractional  
contribution  
to total  
variance*

$1/25$

*Kaiser criterion*

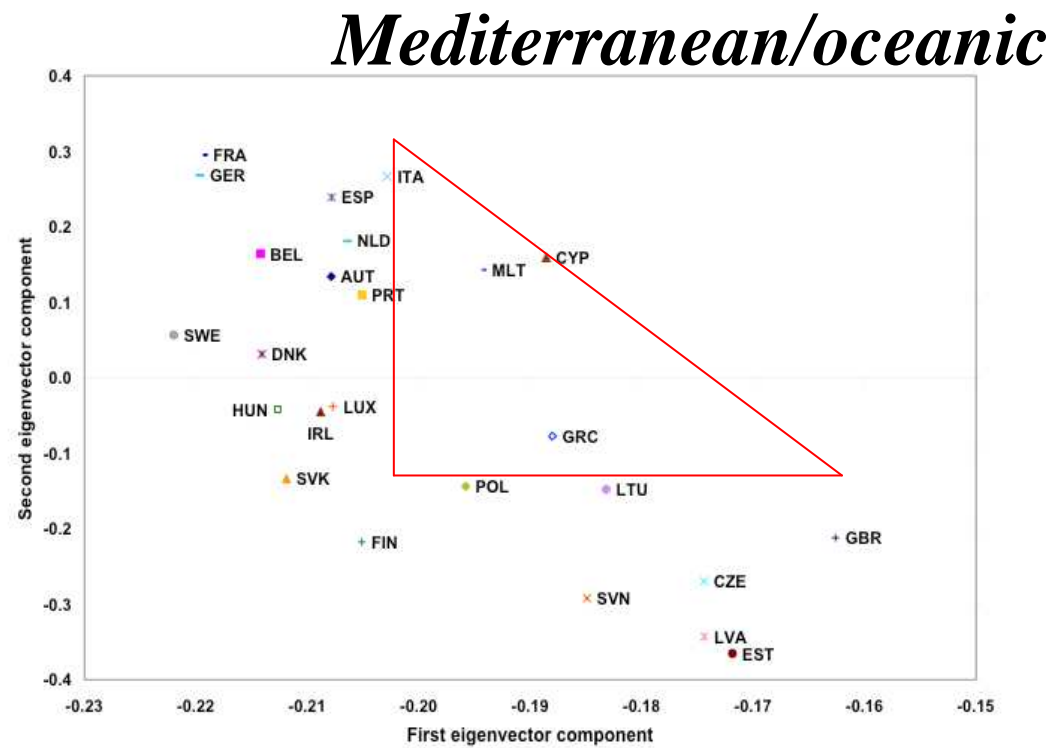
# Cluster structure (1)

*Continental*



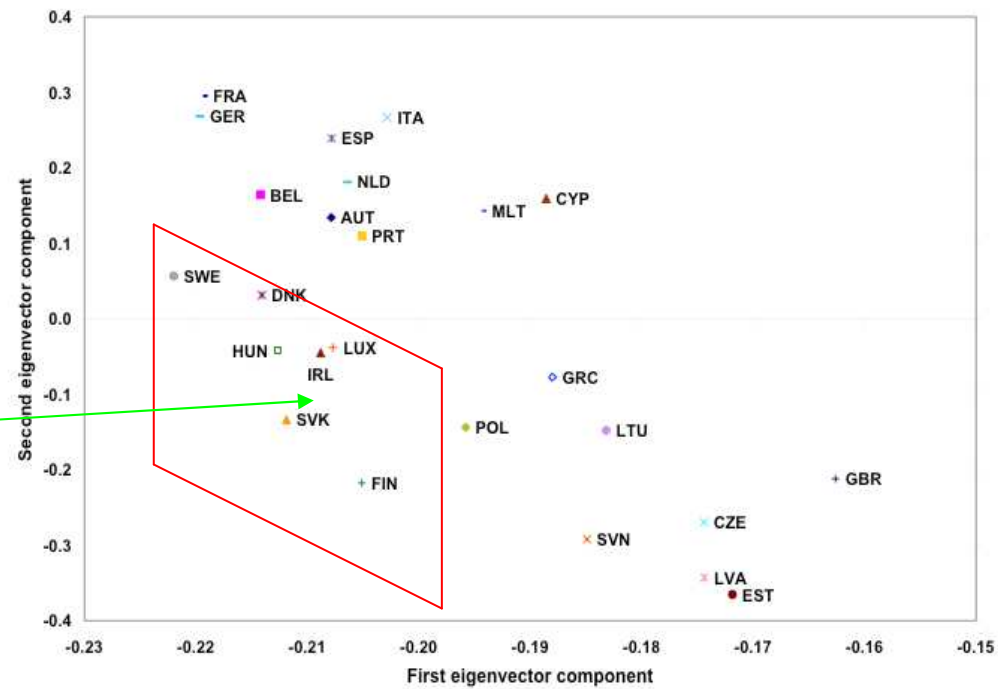
**EU25**

# Cluster structure (2)



**EU25**

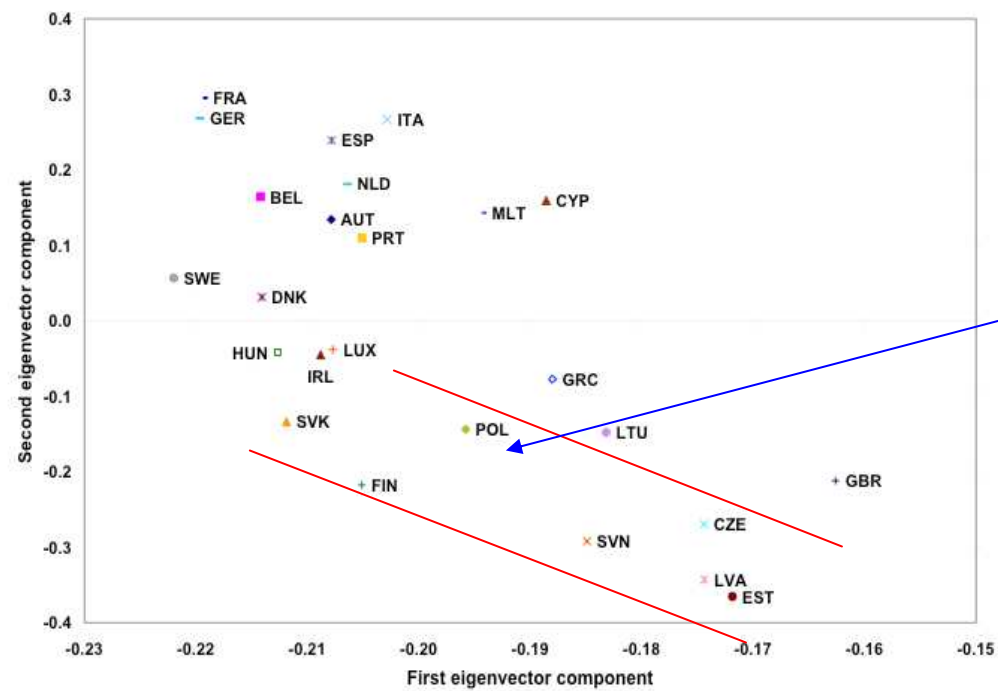
# Cluster structure (3)



**EU25**

*Scandinavian*

# Cluster structure (4)

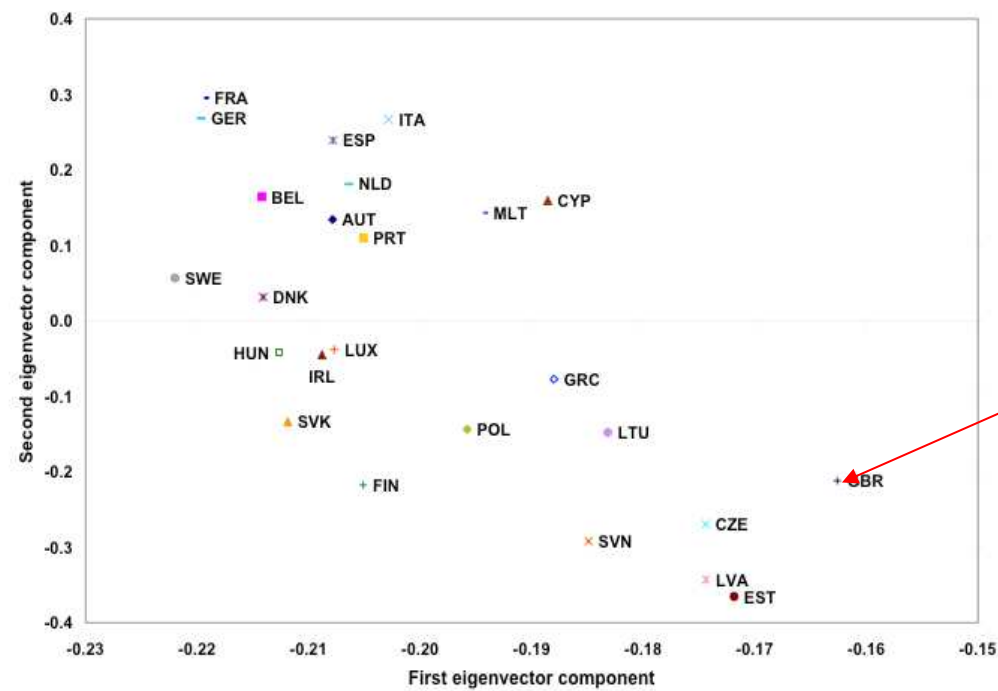


**EU25**

***Central-Eastern***

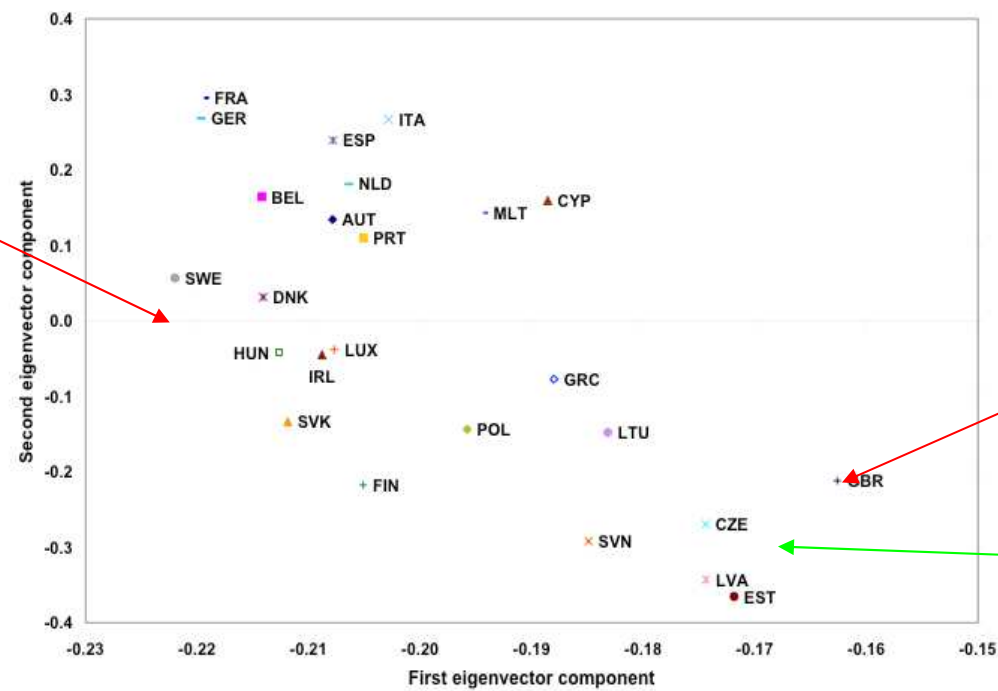
# Cluster structure (5)

EU25





# Cluster structure (5)



# Fisher-z

$$C_{ij}^{(z)}(T) = \sigma \langle z_{ij} \rangle + \mu$$

**Table 1** The first 10 eigenvalues of the correlation matrices constructed by averaging the coefficients of determination (the first row) and Fisher z-values (the second row)

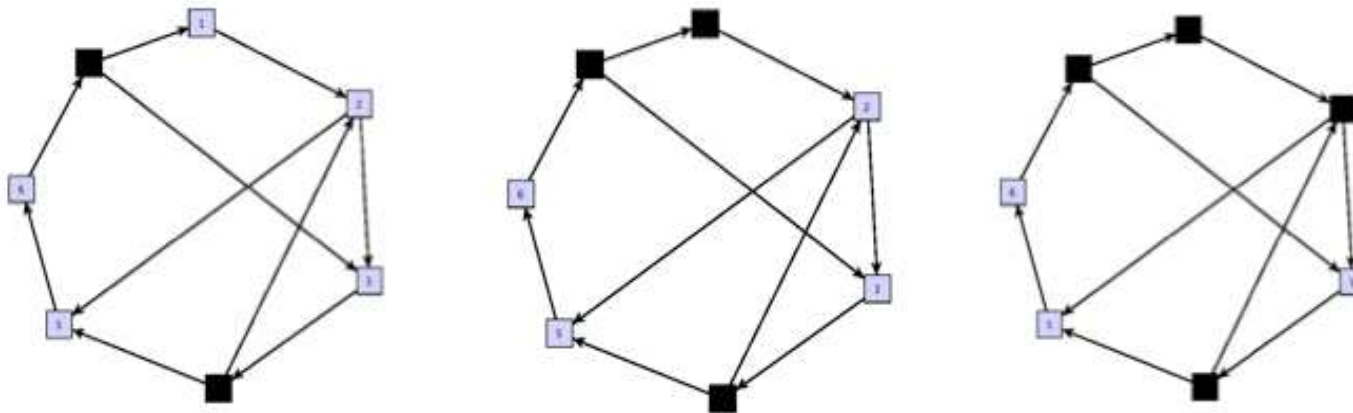
Eval[ $C_y^{(d)}$ ]	<b>15.132</b>	<b>2.255</b>	1.159	1.077	0.912	0.719	0.663	0.603	0.505	0.428
Eval[ $C_y^{(z)}$ ]	<b>10.029</b>	<b>3.813</b>	<b>2.365</b>	<b>2.040</b>	1.412	1.206	0.976	0.860	0.676	0.397

$$z_{ij} = \frac{C_{ij} - \mu}{\sigma}$$

$$\langle z_{ij} \rangle = \frac{1}{\nu} \sum_{i=1}^{k+T} z_i(t)$$

# Model (1)

In a network composed of  $N$  nodes with directed links. Initially, a fraction of the nodes are activated. A node becomes activated at a time step if all the nodes whose links arrive at him were activated.

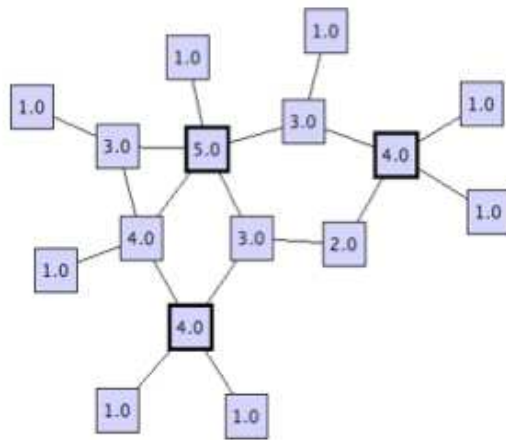


# Model (5)

## Local leadership

Global leaders are certainly important nodes in a network (extreme statistics), but, at the local level, one expects that local leaders may have an important role

Local leaders = nodes whose degree is larger or equal to the degree of all of their neighbours



Probability for a node of degree  $k$  to be a leader:

$$Z_k \rightarrow \begin{cases} 1 & \text{for } \gamma > 3, \\ e^{-1/\langle k \rangle} & \text{for } \gamma = 3, \\ 0 & \text{for } \gamma < 3. \end{cases}$$

$$n_k \approx k^{-\gamma}$$