U. Leiden, Feb. 22, 2010

Exercice #2

DFA: Detrended Fluctuation Analysis

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Motivation

- in order to understand how strong are Long Range and Short Range fluctuations
- to suggest some investment technique

Get rich ? (à la Ausloos)

- obtain the power law behavior describing the rms. deviation of the fluctuations
- as a function of the window size
  - see α value and range of windows

- calculate the time dependence of α exponent
  - moving a window with appropriate size along the time axis, supposing stationarity of the increments !!!
- invent some investment strategy
- calculate the returns (gains, of course)

DEFINITIONS

- ***"Variability"***

$$D_{\Delta t} y(t) = y(t+\Delta t) - y(t)$$

- ***"Log returns"***

$$L\,Z(t\,,\Delta t) = \log[\, y(t+\Delta t)\,/y(t)\,]$$

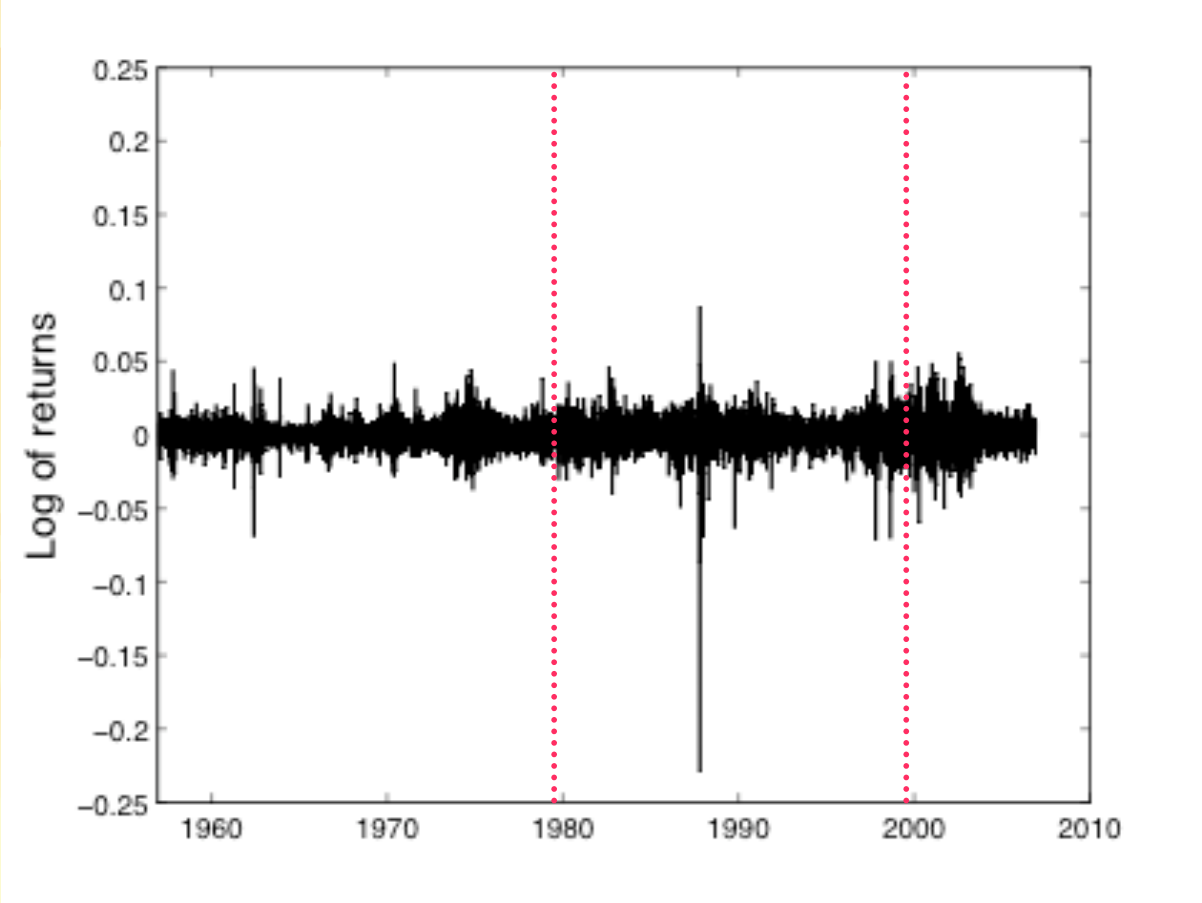
- ***"Normalized variability"***

$$Z(t,\Delta t) = [D_{\Delta t}\, y(t) - \langle D_{\Delta t}\, y\rangle]\,/s_{\Delta t}$$

Time lag effect ?

Standard & Poor 500

http://finance.yahoo.com



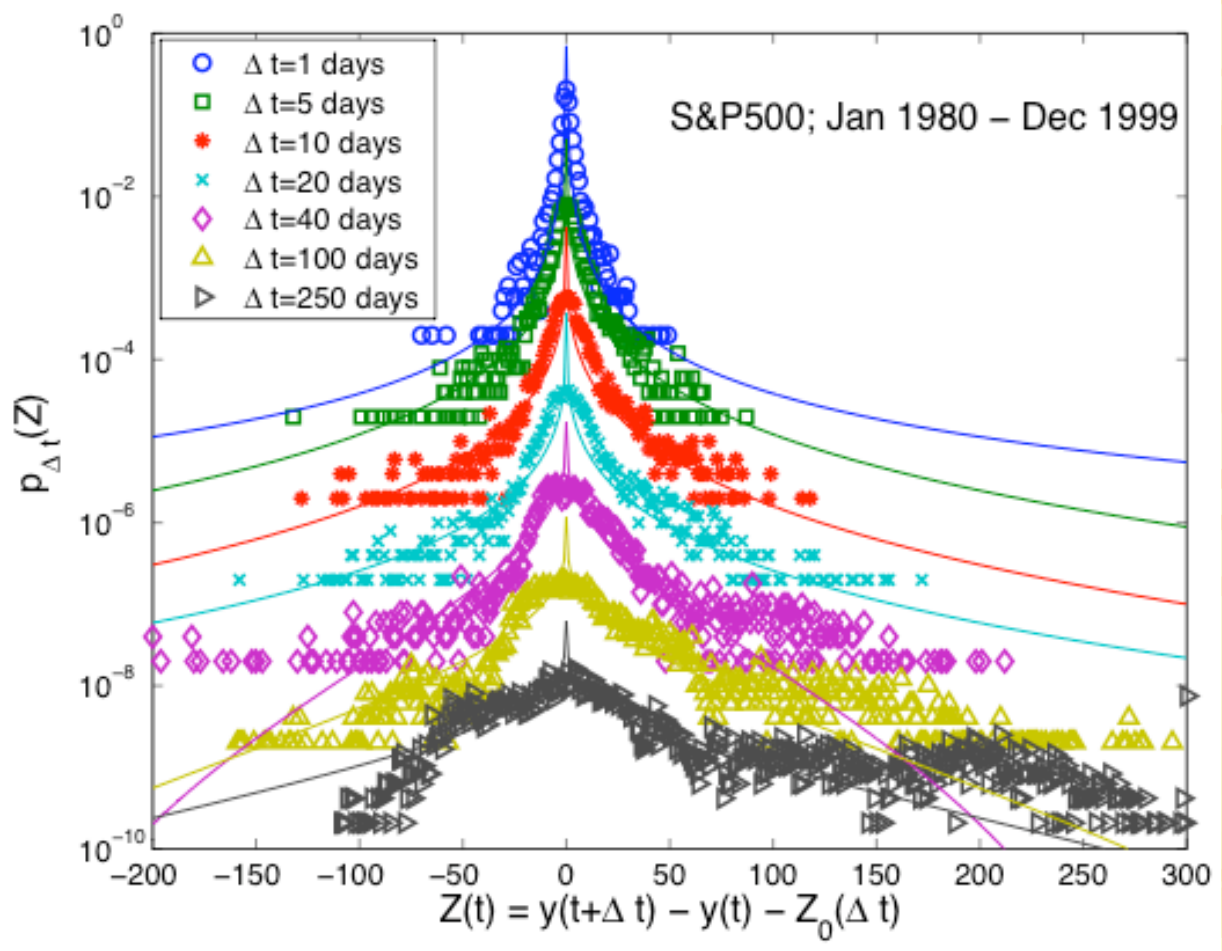
Daily data

(normalized)

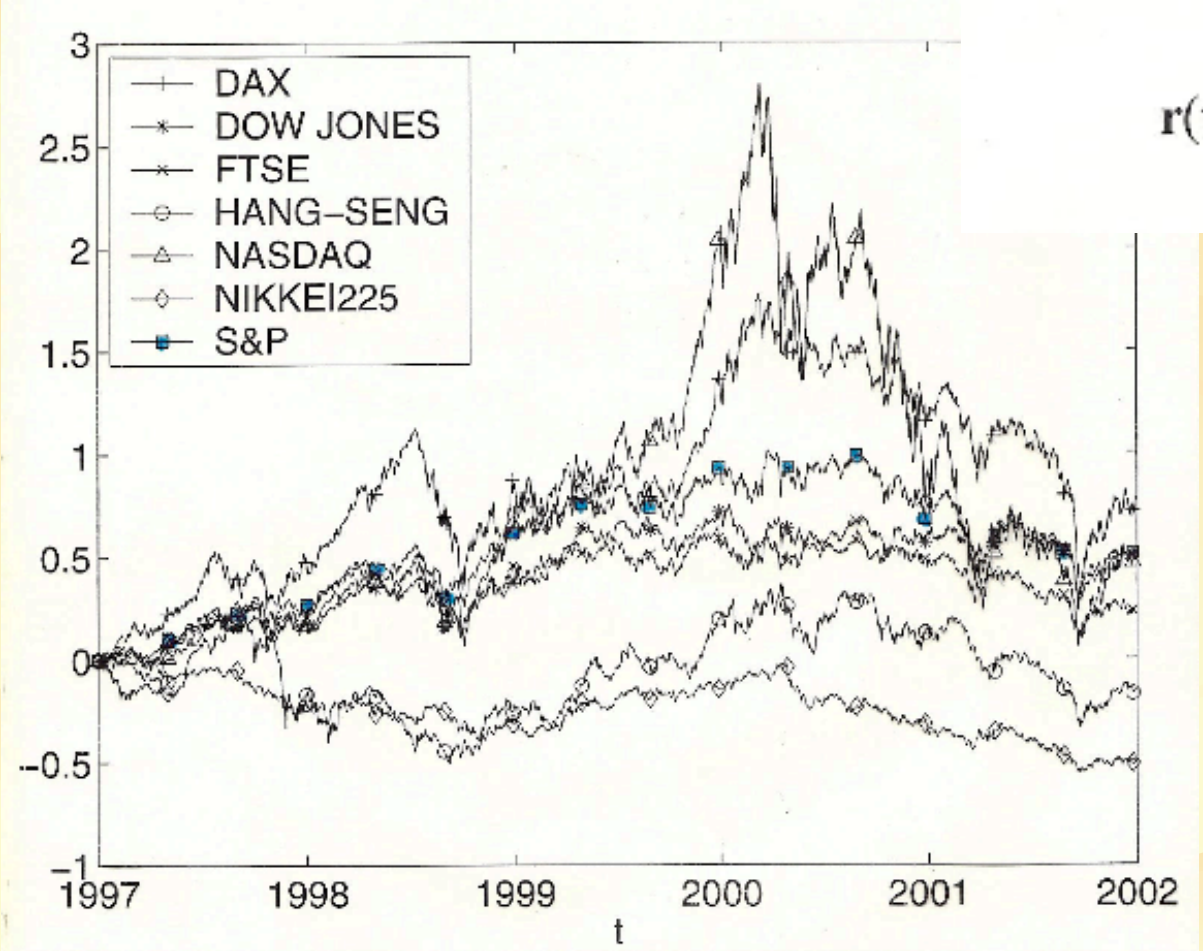
Jan. 01,1980
- Dec. 31,1999

5056
data points

S&P500 *PDF returns*



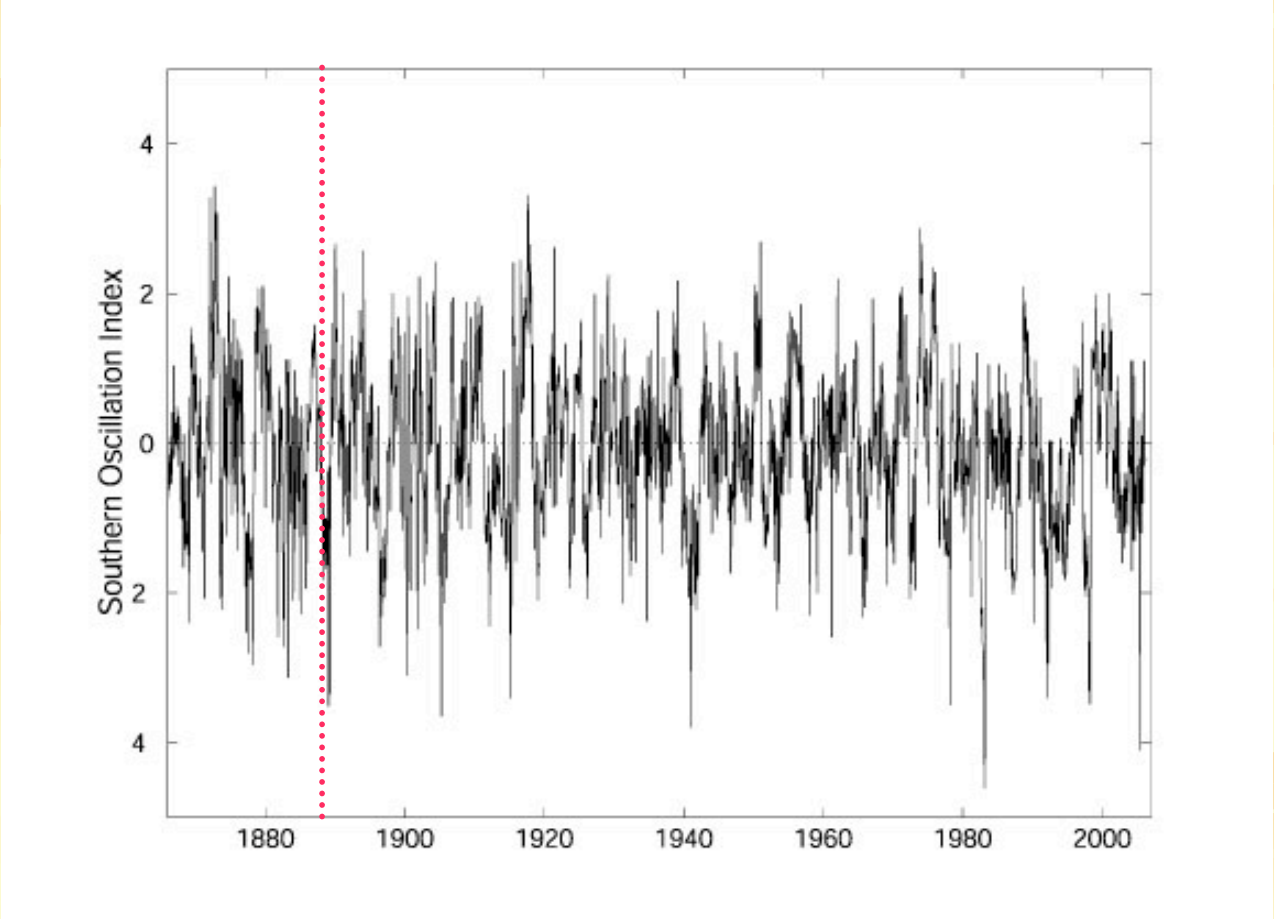
Returns 97-02



$$r(t) = \frac{\Pi(t) - \Pi(t_0)}{\Pi(t_0)}$$

Southern Oscillation Index

http://www.cpc.ncep.noaa.gov/data/indices/



P (Darwin)

-

P (Tahiti) **=**

(normalized)

Monthly data

1886-2006

1672

data points

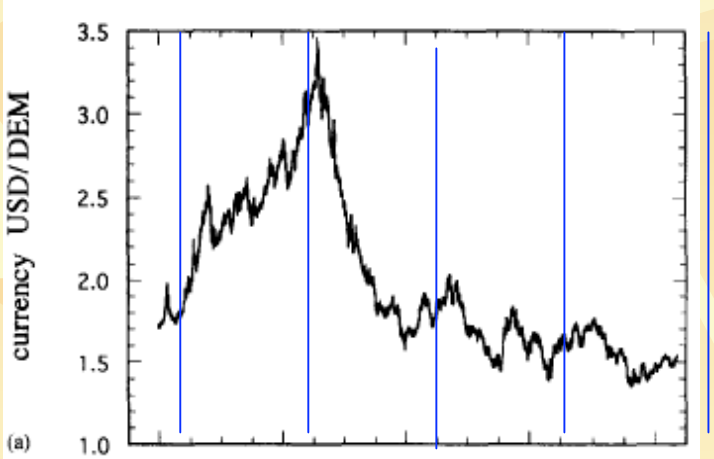
Detrended Fluctuation Analysis

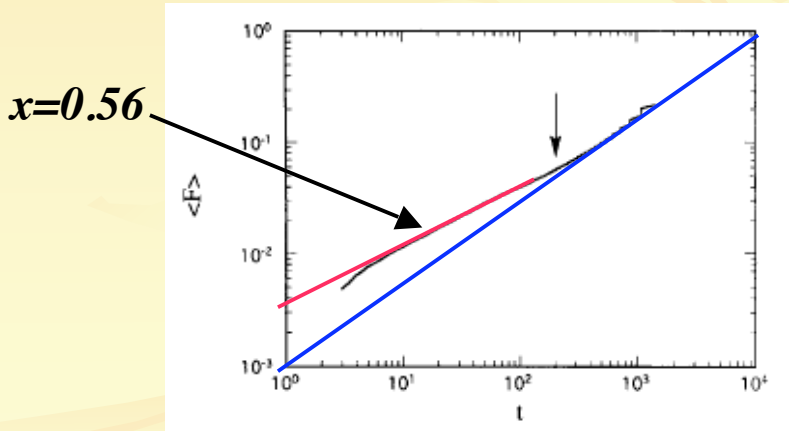
- *(integrated series or not?)*

$$F(t)^2 = \frac{1}{t}\sum_{n=kt+1}^{(k+1)t}(y(n)-z(n))^2$$

$$k = 0, 1, 2, \ldots, \left(\frac{N}{t} - 1\right)$$

$$< F^2(t) >^{1/2} \sim t^{\alpha}$$





DFA web

Software for DFA

The file dfa.c is the C language source for a program that performs detrended fluctuation analysis of a time series. Read about how to use this program here (or download this information in Unix man page format here).

The instructions below assume that you already have a C compiler, such as gcc, and a make utility, such as GNU make. Most GNU/Linux and Unix systems have these already. Under MS-Windows,we recommend the versions of gcc and make included in the free Cygwin development environment; under Mac OS X, use the versions included in Apple's XCode tools.

- Download dfa.c, Makefile, a sample input file (rr-intervals), and the corresponding output file (rr-dfa).
- Build dfa by typing:

```
make
```

- Test the compiled dfa by typing:

```
make check
```

- If dfa passes the test, install it by typing:

```
make install
```

If you wish to use some other C compiler, compile dfa.c and link it with the standard C math library, using whatever method is standard for your C compiler. See Makefile to see how to test the executable file that you compile.

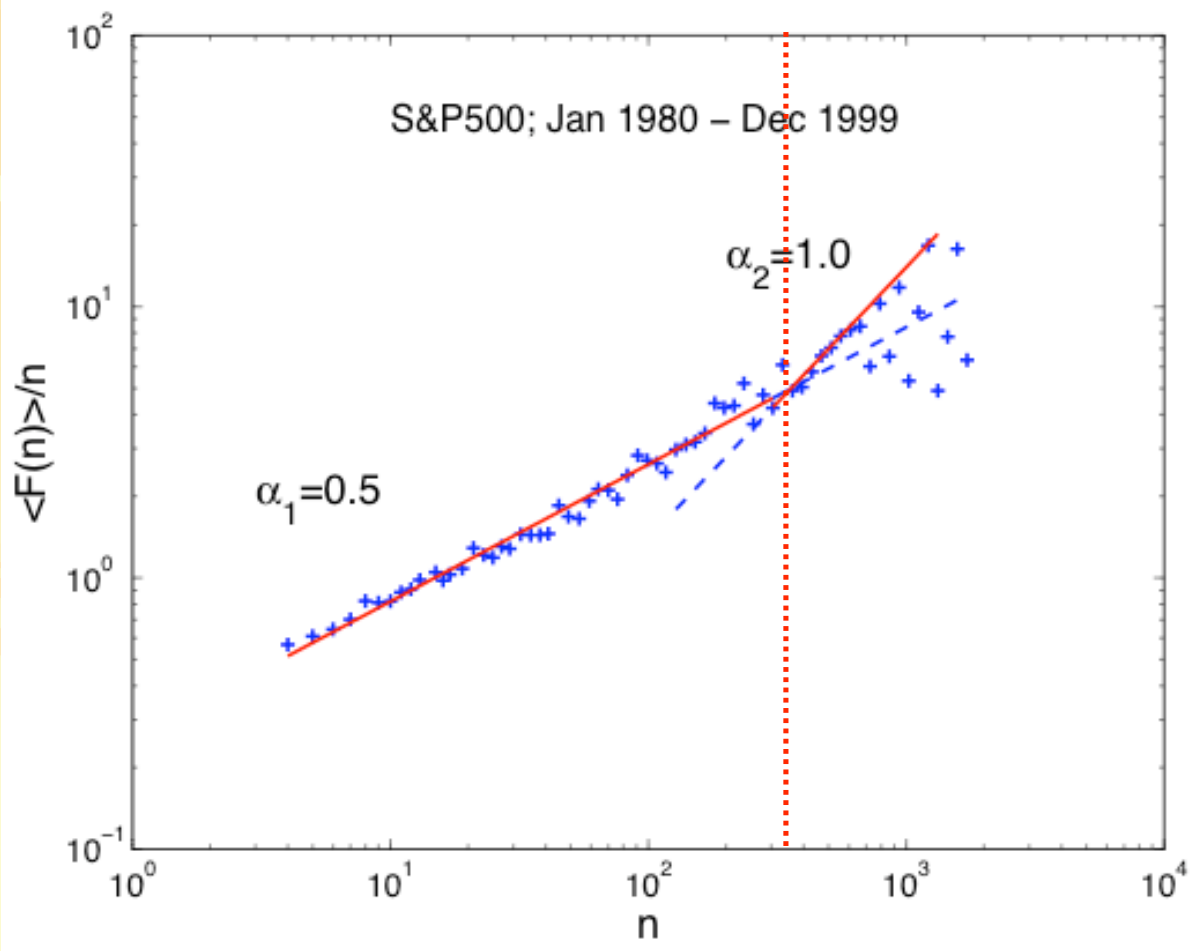
This method was first proposed in: Peng C-K, Buldyrev SV, Havlin S, Simons M, Stanley HE, Goldberger AL. Mosaic organization of DNA nucleotides. *Phys Rev E* 1994;**49**:1685-1689.

A detailed description of the algorithm and its application to physiologic signals can be found in: Peng C-K, Havlin S, Stanley HE, Goldberger AL. Quantification of scaling exponents and crossover phenomena in nonstationary heartbeat time series. *Chaos* 1995;5:82-87.

Please cite at least one of the above publications when referencing this material, and also include the standard citation for PhysioNet:

Goldberger AL, Amaral LAN, Glass L, Hausdorff JM, Ivanov PCh, Mark RG, Mietus JE, Moody GB, Peng C-K, Stanley HE. PhysioBank, PhysioToolkit, and PhysioNet: Components of a New Research Resource for Complex Physiologic Signals. *Circulation* **101**(23):e215-e220 [Circulation Electronic Pages; http://circ.ahajournals.org/cgi/content/full/101/23/e215]; 2000 (June 13).

Detrended Fluctuation Analysis

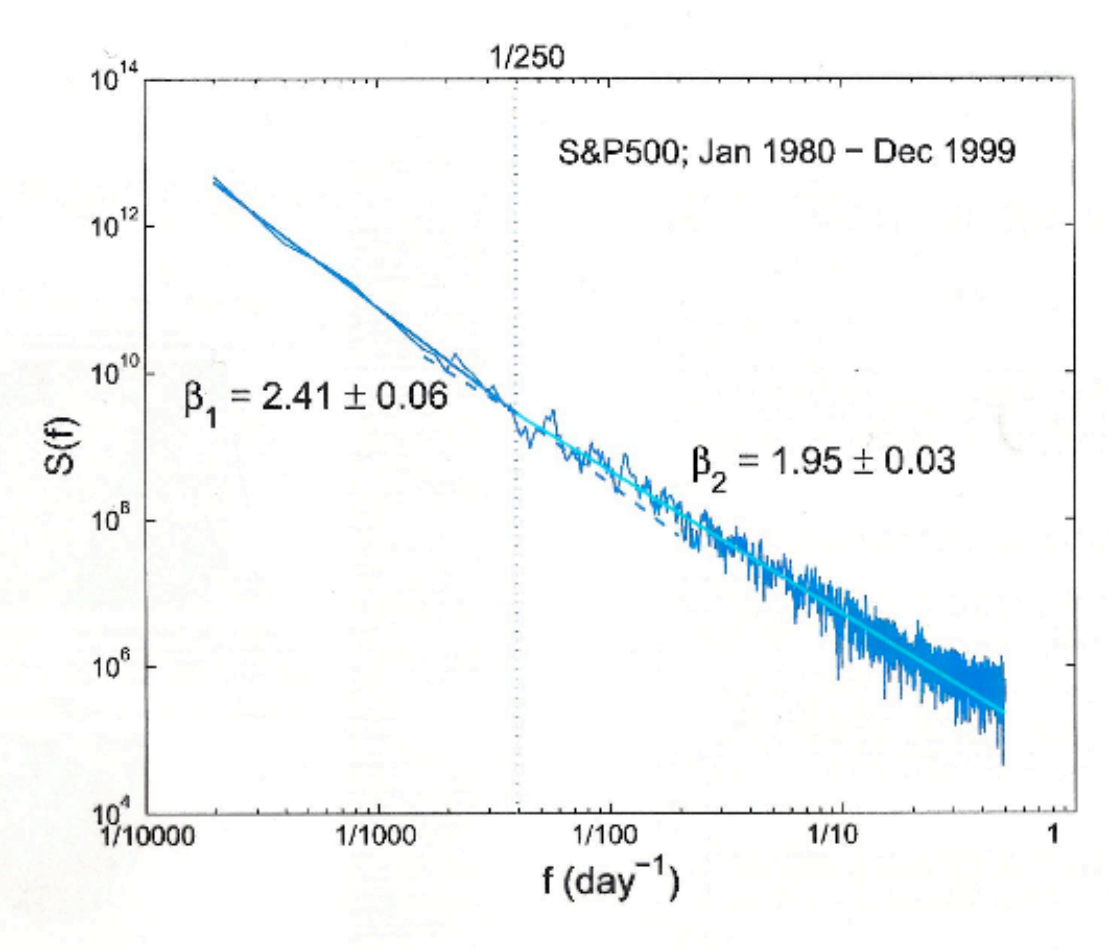


on SP500 increments

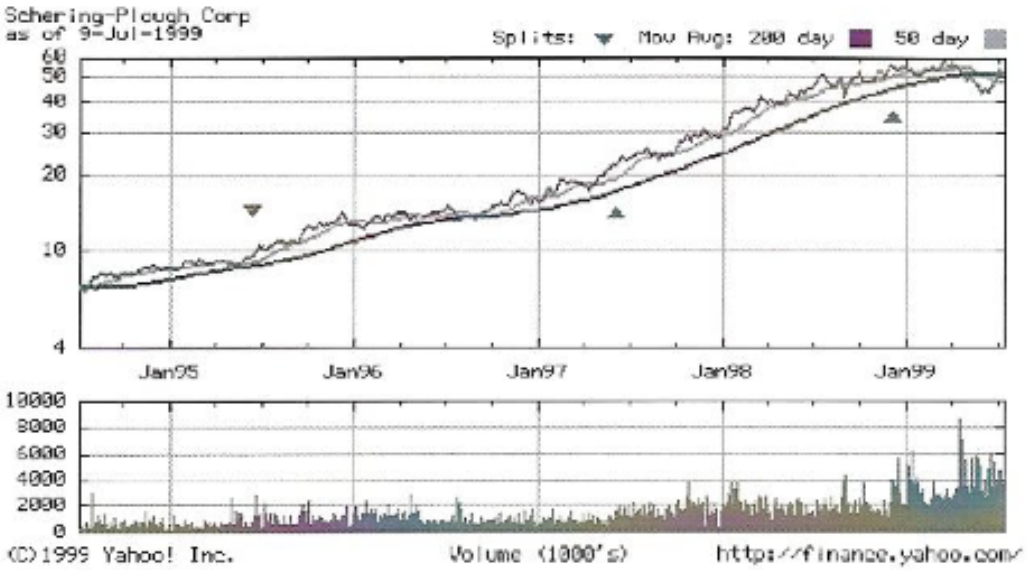
Long Range Correlations ?

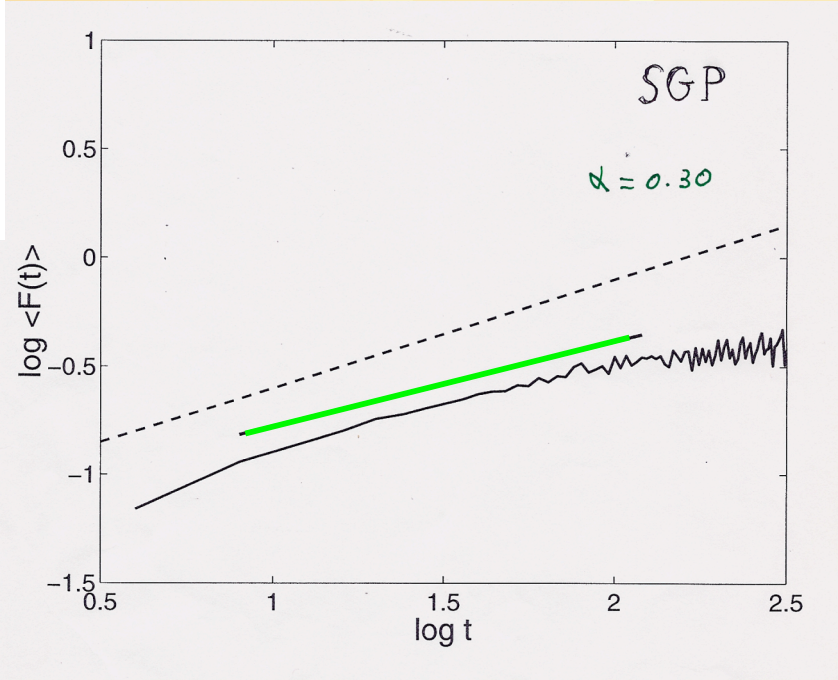
Brownian values

SP500 80-99 β

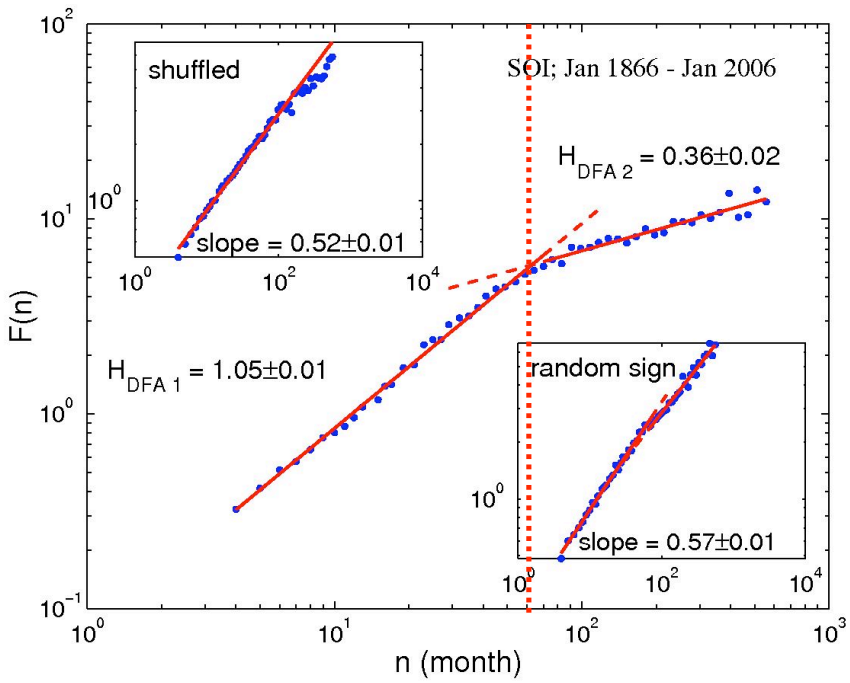

1/250
S&P500; Jan 1980 – Dec 1999
$\beta_1 = 2.41 \pm 0.06$
$\beta_2 = 1.95 \pm 0.03$
S(f)
10^{14}
10^{12}
10^{10}
10^{8}
10^{6}
10^{4}
1/10000
1/1000
1/100
1/10
1
f (day^{-1})

SGP


Schering-Plough Corp
as of 9-Jul-1999
Splits:
Mov Avg: 200 day
50 day
60
50
40
30
20
10
4
Jan95
Jan96
Jan97
Jan98
Jan99
10000
8000
6000
4000
2000
0
(C)1999 Yahoo! Inc.
Volume (1000's)
http://finance.yahoo.com/


SGP
α = 0.30
log <F(t)>
log t
1
0.5
0
−0.5
−1
−1.5
0.5
1
1.5
2
2.5

Detrended Fluctuation Analysis



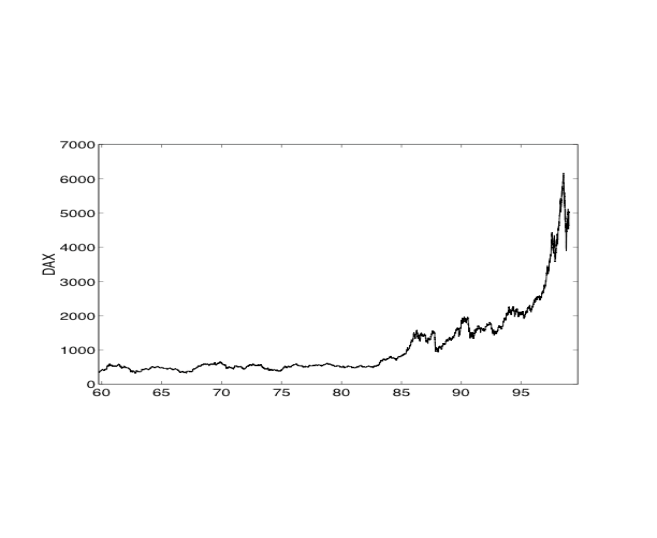
on SOI increments

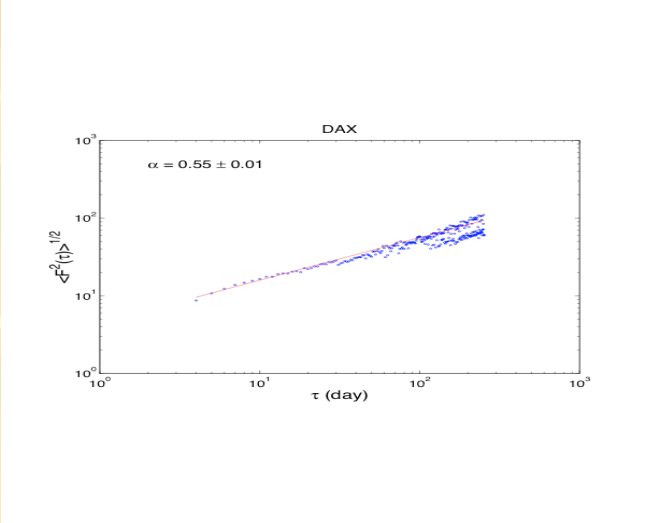
Long Range Correlations ?

Non Brownian values

Polish group DFA for crashes

DAX DFA(q)





Variants/generalizations

+ DFA$_q$

+ DFA$_{q,m}$

$$F_{q,m}(T) = [\frac{1}{T}\sum_{n=kT+1}^{(k+1)T}(y(n) - z_m(n))^q]^{1/q}$$

DFA(q)
word length (L); word frequency (F)

Textes	DFA-1	Pente 0-200	Erreur	200-5000	Erreur	0-5000	Erreur	DFA-2	Pente 0-200	Erreur	200-5000	Erreur	0-5000	Erreur
LTS														
English		0.538	2E-3	0.686	1E-3	0.684	1E-3		0.52	1E-3	0.665	1E-3	0.663	1E-3
Esperanto		0.516	2E-3	0.619	2E-3	0.62	2E-3		0.492	2E-3	0.706	2E-3	0.704	2E-3
Looking glass		0.531	2E-3	0.56	1E-3	0.56	2E-3		0.511	1E-3	0.542	1E-3	0.543	1E-3
English shuffle		0.461	2E-3	0.584	1E-3	0.581	1E-3		0.472	2E-3	0.617	2E-3	0.611	2E-3
Esperanto shuffle		0.519	4E-3	0.507	1E-3	0.506	1E-3		0.526	3E-3	0.461	2E-3	0.464	2E-3
Looking glass shuffle		0.504	3E-3	0.587	1E-3	0.584	1E-3		0.513	2E-3	0.581	1E-3	0.579	1E-3
FTS														
English		0.491	2E-3	0.561	2E-3	0.561	2E-3		0.463	1E-3	0.585	1E-3	0.584	1E-3
Esperanto		0.519	2E-3	0.544	1E-3	0.545	1E-3		0.482	2E-3	0.47	1E-3	0.475	1E-3
Looking glass		0.501	2E-3	0.777	3E-3	0.774	3E-3		0.471	1E-3	0.7	1E-3	0.695	1E-3
English shuffle		0.525	1E-3	0.534	1E-3	0.533	1E-3		0.515	1E-3	0.568	1E-3	0.566	1E-3
Esperanto shuffle		0.518	1E-3	0.474	1E-3	0.478	1E-3		0.517	2E-3	0.455	1E-3	0.460	1E-3
Looking glass shuffle		0.524	1E-3	0.480	1E-3	0.480	1E-3		0.535	1E-3	0.506	1E-3	0.505	1E-3

	DFA-3	Pente 0-200	Erreur	200-5000	Erreur	0-5000	Erreur	DFA-4	Pente 0-200	Erreur	200-5000	Erreur	0-5000	Erreur
LTS														
English		0.514	1E-3	0.678	2E-3	0.675	2E-3		0.511	2E-3	0.667	2E-3	0.664	2E-3
Esperanto		0.475	2E-3	0.759	2E-3	0.755	2E-3		0.4655	2E-3	0.662	1E-3	0.659	1E-3
Looking glass		0.506	1E-3	0.57	1E-3	0.569	1E-3		0.507	1E-3	0.558	1E-3	0.558	1E-3
English shuffle		0.4853	2E-3	0.56	2E-3	0.556	2E-3		0.499	2E-3	0.474	1E-3	0.474	1E-3
Esperanto shuffle		0.529	2E-3	0.487	1E-3	0.461	1E-3		0.524	2E-3	0.469	1E-3	0.472	1E-3
Looking glass shuffle		0.517	1E-3	0.534	1E-3	0.533	1E-3		0.519	2E-3	0.553	1E-3	0.551	1E-3
FTS														
English		0.453	1E-3	0.621	1E-3	0.618	1E-3		0.448	2E-3	0.547	1E-3	0.546	1E-3
Esperanto		0.46	1E-3	0.507	1E-3	0.51	1E-3		0.452	2E-3	0.522	1E-3	0.524	1E-3
Looking glass		0.457	2E-3	0.676	1E-3	0.673	1E-3		0.449	1E-3	0.656	2E-3	0.652	2E-3
English shuffle		0.504	2E-3	0.545	1E-3	0.543	1E-3		0.500	2E-3	0.519	1E-3	0.519	1E-3
Esperanto shuffle		0.509	2E-3	0.480	1E-3	0.483	1E-3		0.504	2E-3	0.519	1E-3	0.520	2E-3
Looking glass shuffle		0.524	2E-3	0.517	1E-3	0.516	1E-3		0.515	2E-3	0.505	1E-3	0.505	1E-3

DFA forecasting strategy

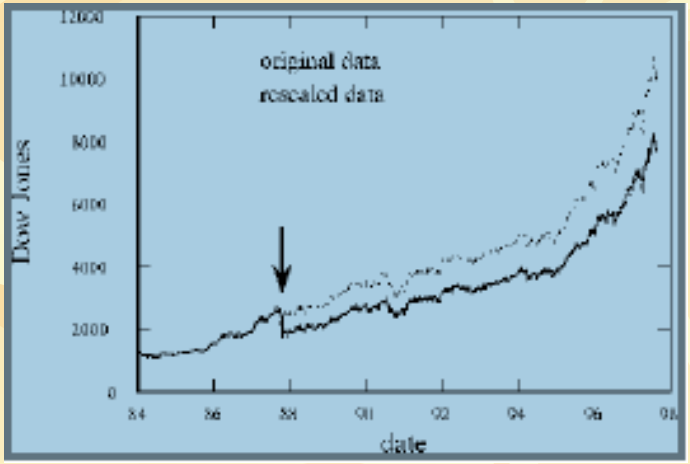
- If $\alpha < 0.5$
- Antipersistence
- Negative correlation
- « very noisy »
- If $\alpha <<<< 0.5$
- up, down, up, down, up, down, up, down,..
- .... probabilistic
- .
- $\alpha = 0$: white noise

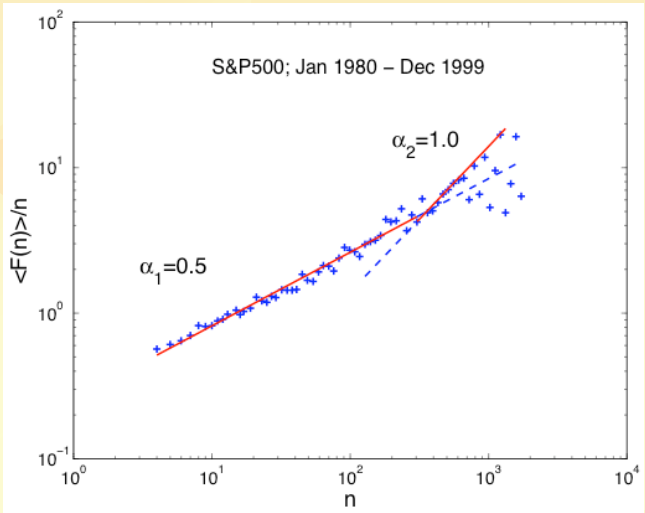
- If $\alpha > 0.5$
- Persistence
- Positive correlation
- « very smooth »
- If $\alpha >>>> 0.5$
- Up, up, up, up, up, ..., down, down, down, ..
- .... probabilistic
- $\alpha = 1$: smooth line

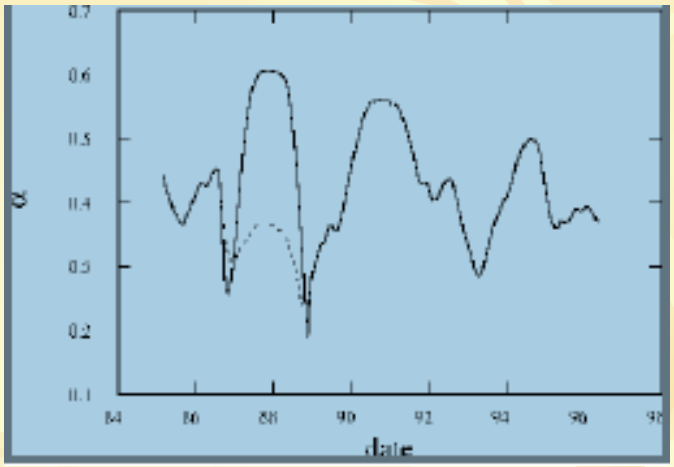
$\alpha = 0.5$ (or $\beta = 2$) : Brownian motion

Local DFA: DJIA Jan.84-Aug.97

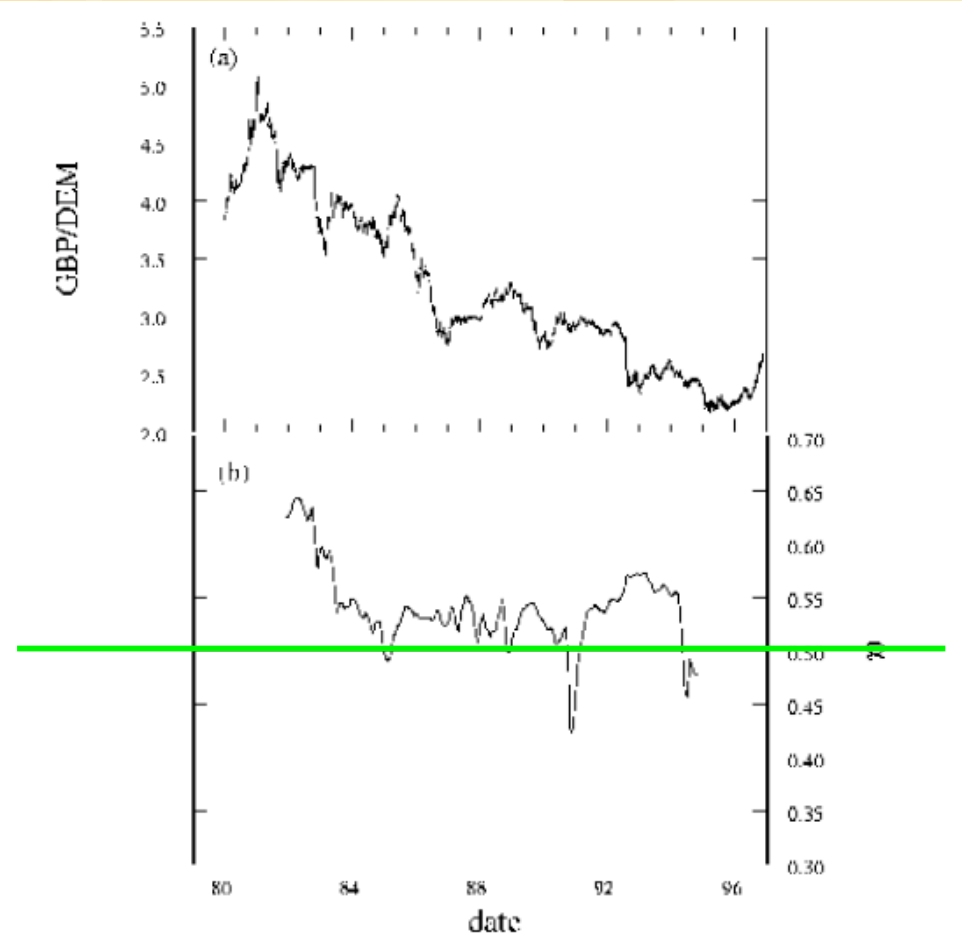
- *"local" (or temporal) DFA*



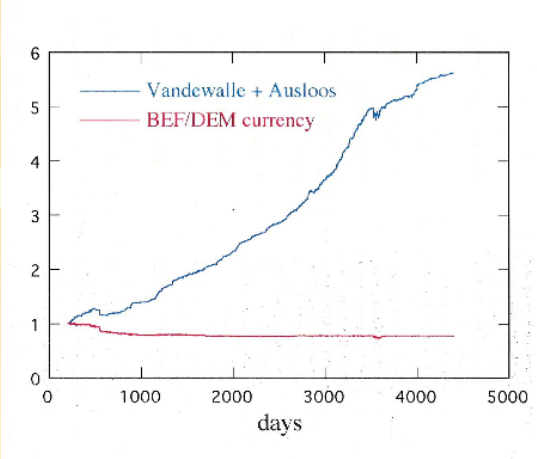


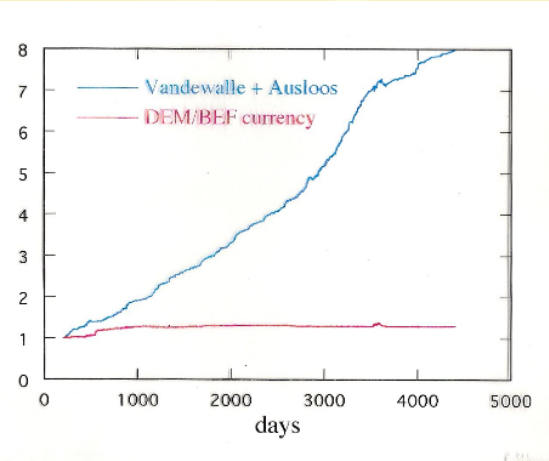


« Local » DFA


(a)
(b)
GBP/DEM
5.5
5.0
4.5
4.0
3.5
3.0
2.5
2.0
0.70
0.65
0.60
0.55
0.50
0.45
0.40
0.35
0.30
80
84
88
92
96
date

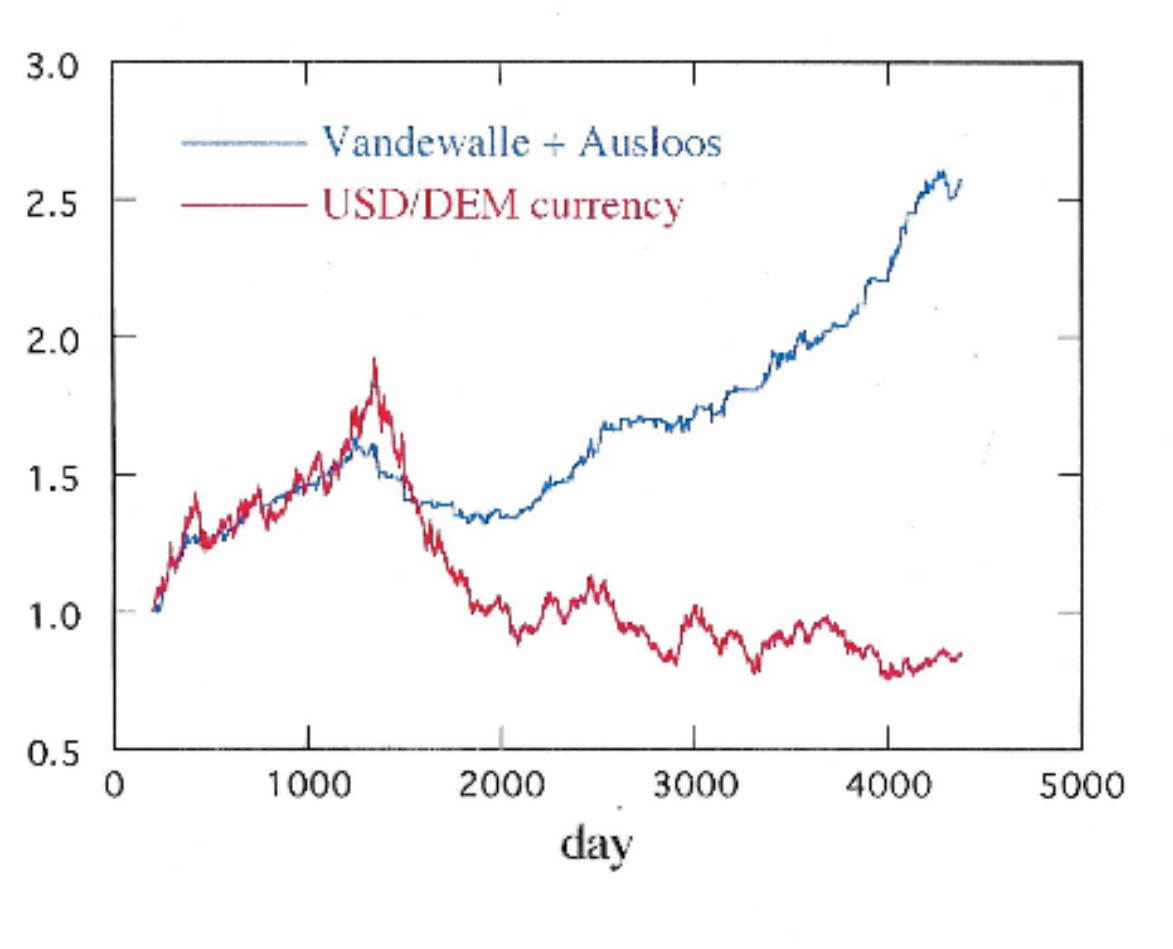
BEF - DEM



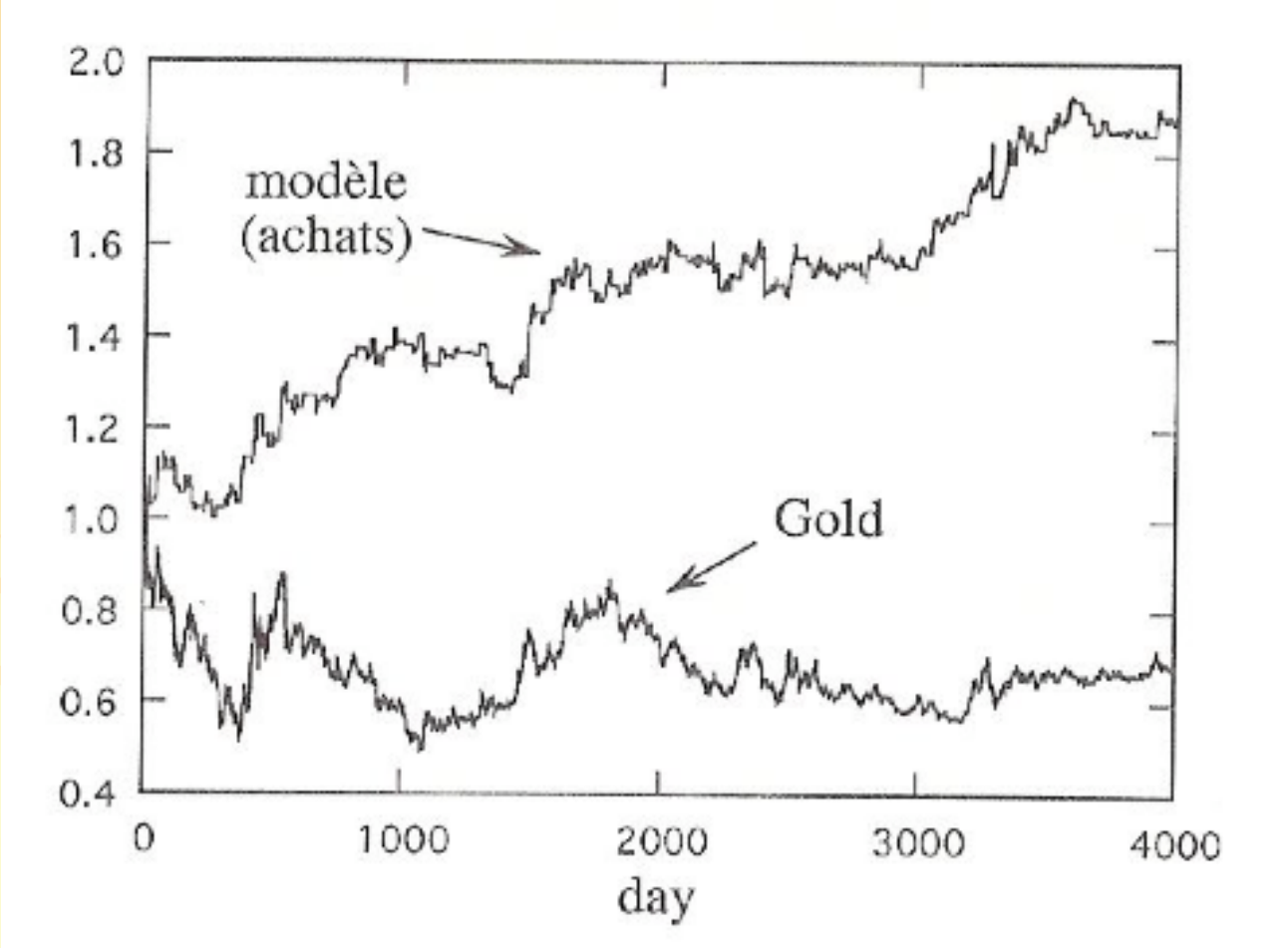


- (very) antipersistent
- $\alpha = 0.23$
- ...
- N.B.
  - NLG/BEF : 0.26
  - DKK/BEF : 0.31
  - FRF/BEF : 0.37
- N.B.
  - JPY/USD : 0.55
  - USD/CAD : 0.55
  - GBP/USD : 0.55
  - USD/DEM : 0.55

USD/DEM gain


3.0
2.5
2.0
1.5
1.0
0.5
0
1000
2000
3000
4000
5000
day
Vandewalle + Ausloos
USD/DEM currency

Gold


2.0
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0
1000
2000
3000
4000
day
modèle
(achats)
Gold

Make money