New particles

- Nuclear physics, two types of nuclear physics phenomena: $\alpha$-decay and $\beta$-decay

- Cosmic rays, first accelerators produced many new particles (positrons, muon, pions, ...)  

- These phenomena did not find their explanation in the framework of QED
Weak & strong interactions

\( \alpha \)-decay

**Strong interactions:**
- Discovery of proton (1919)
- Discovery of neutron (1932)
- Yukawa theory (1935). Prediction of strongly interacting particle with mass \( \sim 100 \text{ MeV}/c^2 \)
- Discovery of pion (1947)

\( \beta \)-decay

**Weak interactions:**
- Measurement of continuous spectrum of electrons in \( \beta \) decay (1927)
- Prediction of neutrino (1930)
- Discovery of positron (1932)
- Fermi theory (1934)
- Discovery of muon (1937)
- Demonstration that muon interacts too weakly for nuclear forces (1947)

See timeline here: hep-ph/0001283
Electromagnetic interactions are mediated by exchange of photons

Yukawa’s idea – proton and neutron exchange some new massive particle to interact in the nuclei


Static $p - n$ interaction was known to decrease rapidly at the distances $\geq 2 \text{ fm} = 2 \times 10^{-13} \text{ cm}$.

The idea of Yukawa – there is $p - n$ interaction of the form

$$U(r) = \frac{g^2}{4\pi r} e^{-r/a} \quad (1)$$

where $a \lesssim 1 \text{ fm}$

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0. This presentation closely follows Aitchison & Hey, Sec. 2.2 (see also Sec. 2.1 and 2.3–2.5)
### Yukawa interaction

\[ U(r) = \frac{g^2}{4\pi r} e^{-r/a} \text{ – short ranged} \]

### Electromagnetic interaction

\[ U(r) = \frac{e^2}{4\pi r} \text{ – long ranged} \]

#### Poisson equation

\[ \left( \nabla^2 - \frac{1}{a^2} \right) U(r) = g^2 \delta(r) \]

\[ \nabla^2 U(r) = e^2 \delta(r) \]

#### Free-space propagation

\[ \left( \nabla^2 - \frac{1}{c^2 \partial t^2} - \frac{1}{a^2} \right) U(r) = 0 \]

\[ \left( \nabla^2 - \frac{1}{c^2 \partial t^2} \right) U(r) = 0 \]

#### De Broglie solution

\[ U \propto \exp \left( \frac{ip \cdot x}{\hbar} - \frac{iEt}{\hbar} \right) \text{ implies that} \]

\[ E^2 = p^2 c^2 + \frac{c^2 \hbar^2}{a^2} \]

\[ \text{Mass of quanta : } m_Y \equiv \frac{\hbar}{ac} \]

\[ E^2 = c^2 p^2 \]

\[ \text{Mass of quanta} = 0 \]
Massive Klein-Gordon equation

- **Build** the Green’s function of the Klein-Gordon equation

\[(\Box + m^2)\phi(x) = \delta^{(4)}(x - x')\]

- use \(m^2 \rightarrow m^2 - i\epsilon\) to obtain **causal Green’s function**

- **Find** its spatial representation. Consider two cases: \((x - x')^\mu = (t, 0)\) and \((x - x')^\mu = (0, \vec{x})\). Find behavior of the Green’s function when \(t \rightarrow \infty\) or \(|\vec{x}| \rightarrow \infty\)
Yukawa theory predicted a particle with the mass $m_Y \approx 100 \text{ MeV}/c^2$ (from $a \approx 2 \text{ fm}$), responsible for proton-neutron (nuclear) interactions.

Therefore an emission process $n \to p + Y$ is impossible for the real particle $Y$ (energy conservation violated).

Exchange by the virtual particle mediates the proton-neutron interaction.
Weak & strong interactions

$\alpha$-decay

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$\beta$-decay

Weak interactions:
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See timeline here: hep-ph/0001283
Fermi theory of $\beta$–decay

- Neutron decay $n \rightarrow p + e^- + \bar{\nu}_e$

- Two papers by E. Fermi:
  - *An attempt of a theory of beta radiation. 1.* (In German) Z.Phys. 88 (1934) 161-177
    DOI: 10.1007/BF01351864
  - *Trends to a Theory of beta Radiation.* (In Italian) Nuovo Cim. 11 (1934) 1-19
    DOI: 10.1007/BF02959820

- Fermi 4-fermion theory:

  $$\mathcal{L}_{\text{Fermi}} = -\frac{G_F}{\sqrt{2}} \left[ \bar{p}(x) \gamma_\mu n(x) \right] \left[ \bar{e}(x) \gamma^\mu \nu(x) \right]$$  \hspace{1cm} (2)

- New phenomenological constant, $G_F$, **Fermi constant.**

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$^1$History of $\beta$-decay (see [hep-ph/0001283], Sec. 1.1); Cheng & Li, Chap. 11, Sec. 11.1)
Fermi theory of $\beta$-decay

- Dimension of the Fermi constant? .
  Mass operator $M\bar{\psi}\psi \Rightarrow [\bar{\psi}\psi] = M^3 \implies [G_F] = M^{-2}$

- Value determined experimentally to be $G_F \approx 10^{-5}$ GeV$^{-2}$ (More precisely $\frac{G_F}{(\hbar c)^3} = 1.166 \times 10^{-5}$ GeV$^{-2}$.)

- Fermi Lagrangian includes leptonic and hadronic terms:

  \[
  \mathcal{L}_{\text{Fermi}} = -\frac{G_F}{\sqrt{2}} \left[ J^\dagger_{\text{lepton}}(x) + J^\dagger_{\text{hadron}}(x) \right] \cdot \left[ J_{\text{lepton}}(x) + J_{\text{hadron}}(x) \right] \tag{3}
  \]

- Lagrangian (3) predicts universality of weak interactions: all processes in Fermi theory can be described in terms of only one constant, $G_F$

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Fermi theory predicts **lepton-only** weak interactions, such as $e + \nu_e \rightarrow e + \nu_e$ scattering

$$\mathcal{L}_{\nu e} = \frac{4G_F}{\sqrt{2}} (\bar{e}\gamma^\nu e)(\bar{\nu}_e\gamma^\lambda e)$$ (4)

Matrix element for $e + \nu_e \rightarrow e + \nu_e$ scattering $|\mathcal{M}|^2 = \left| \langle e^-, \nu_e | \mathcal{L}_{\text{Fermi}} | e^-, \nu_e \rangle \right|^2$ (summed over spins of outgoing particles and averaged over spins of initial particles)

$$\sum_{\text{spins}} |\mathcal{M}|^2 \propto \left| G_F \bar{u}(p_e)\gamma^\lambda u(p_{\nu}^{\text{in}})\bar{u}(p_{\nu})\gamma^\lambda u(p_e^{\text{in}}) \right|^2$$

$$\propto G_F^2 (p_e^{\text{in}} \cdot p_{\nu}^{\text{in}})(p_{\nu} \cdot p_e) \propto G_F^2 E_{\text{c.m.}}^4$$ (5)
Neutrino-electron scattering

- center-of-mass energy of the system is
  \[ E_{c.m.}^2 = \frac{1}{4}(p_{e \text{ in}}^\text{in} + p_{\nu}^\text{in})^2 \approx (p_{e \text{ in}}^\text{in} \cdot p_{\nu}^\text{in}) \] for \( E_{c.m.} \gg m_e \)
- momentum of one of the incoming particles \( p_{c.m.} \approx E_{c.m.} \) for \( E_{c.m.} \gg m_e \)

- Cross-section:

\[
d\sigma = \frac{1}{4|p_{c.m.}|E_{c.m.}(2E_e)(2E_\nu)} |\mathcal{M}|^2 \delta(E_{c.m.} - E_e - E_\nu)\delta^3(p_e + p_\nu) \tag{6}
\]

- **Reproduce** this calculation using the Lagrangian (4). Keep in mind that neutrino is a purely massless two-component spinor, i.e.

\[
\gamma^\mu p_\mu (1 + \gamma_5) \nu(p) = 0
\]

where \( \frac{1}{2}(1 + \gamma_5) \) is the projector that selects only two (left) component of any 4-component spinor

- Show that any solution of the Dirac equation for neutrino depending only on \( t \) and one spatial coordinate \( (z) \) is “left-moving” (i.e. the wave-function has the general form \( \psi(t, z) = f(t + z) \))
Low energy weak processes

- Muon ($\mu^-$) discovered in cosmic rays (charge of the electron, mass $\approx 100$ MeV) was first confused with the meson (predicted by Yukawa theory). Originally called $\mu$-meson.

- Later $\pi$ mesons and other mesons were discovered. They were all similar, but $\mu^-$ was not interacting with nuclear forces as any meson.

- So muon turned out to be just a “heavy electron”.

- Fermi Lagrangian (3) was extended so that leptonic current $J_{\text{lepton}}$ would include muon:

$$J_{\text{lepton}}^\lambda = \bar{\nu}_e \gamma^\mu e + \bar{\nu}_\mu \gamma^\lambda \mu$$

- …describing for example, the muon decay

$$\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu$$
Low energy weak processes

implies the existence of muon neutrino $\nu_\mu$
discovered in 1962

- Muon mass $m_\mu \approx 105\text{ MeV}$. $m_\mu \gg m_e$. In the rest frame of muon $e^-$, $\nu_e$, $\nu_\mu$ can be taken as approximately massless.\(^5\)

- The answer should be constructed as a product of two dimensionful quantities. Can construct the only object with dimension of time, proportional to $G_F^{-2}$.\(^6\)

\[
\text{Decay width } \Gamma_\mu \propto G_F^2 m_\mu^5 \text{ some powers of } \pi \ldots
\]

- **Find** the exact answer: $\Gamma = \frac{G_F^2 m_\mu^5}{192\pi^3}$ using the analog of Lagrangian (4) where one $\bar{e}\gamma_\lambda \nu_e$ term is substituted with $\bar{\mu}\gamma_\lambda \nu_\mu$ term

\(^5\)There can be a caveat here! Will see it when analyzing pion decay

\(^6\)Actually, we will talk about decay width with the dimension sec\(^{-1}\) or equivalently, GeV. Recall that decay width of 1 GeV corresponds to the lifetime $6.6 \times 10^{-25}\text{ sec}$!
Notice, that similar argument would not work on the neutron’s decay:

\[ n \rightarrow p + e^- + \bar{\nu}_e \]

as this time \( m_n \approx m_p \) and therefore one cannot simply write \( \Gamma_n \propto G_F^2 m_n^5 \). Rather it should be some combination of \( m_n = 939.5 \text{ MeV} \) and \( (m_n - m_p) \approx 1.3 \text{ MeV} \).
We saw in Eq. (5) that matrix element for $e + \nu_e \rightarrow e + \nu_e$ scattering grows with energy, $|\mathcal{M}|^2 \propto G_F^2 E_{c.m.}^4$.

Unitarity means that this element should be bounded from above: $|\mathcal{M}| \leq \text{const}$

Therefore, the Fermi theory would predict meaningless answers for scattering at energies $E_{c.m.} \gtrsim \sqrt{G_F} \approx 300 \text{ GeV}$

The situation would be ameliorated if $G_F$ were energy dependent and decreasing with energy

Promote point-like 4-fermion Fermi interaction to interaction, mediated by a vector boson: $\mathcal{L}_{\text{Fermi}} \rightarrow \mathcal{L}_W = g(W^\mu J_\mu + h.c.)$
Massive intermediate particle

\[ \langle W_\mu(p)W_\nu(-p) \rangle = -i \frac{\eta_{\mu\nu} - \frac{p_\mu p_\nu}{M_W^2}}{p^2 - M_W^2}; \sigma \sim \frac{E^2}{(E^2 - M_W^2)^2} \]

- Propagator of the massive vector boson:

- When mass \( M_W \gg p \) – reduces to Fermi Lagrangian
Vector boson vs. photon

- Propagator of the massive vector boson:

\[ \langle W_\mu(p)W_\nu(-p) \rangle = -i \frac{\eta_{\mu\nu} - \frac{p_\mu p_\nu}{M_W^2}}{p^2 - M_W^2 + i\epsilon} \]

- Recall: photon propagator: \( \langle A_\mu(p)A_\nu(-p) \rangle = \frac{-i\eta_{\mu\nu}}{p^2 + i\epsilon} \)

- Try to put \( M_W \to 0 \). Will you recover photon-like propagator? **No!**
  Trouble with the term in the numerator