Topics in Standard Model

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New particles

See Introduction of this article for the history

- Nuclear physics, two types of nuclear physics phenomena: α-**decay** and β-**decay**
- Cosmic rays, first accelerators produced many new particles (positrons, muon, pions, . . .)
- These phenomena did not find their explanation in the framework of QED

Weak & strong interactions

α-decay

Strong interactions:

– Discovery of proton (1919)

– Discovery of neutron (1932)

– Yukawa theory (1935). Prediction of strongly interacting particle with mass ~ 100 MeV$/c^2$

– Discovery of pion (1947)

β-decay

Weak interactions:

– Measurement of continuous spectrum of electrons in β decay (1927)

– Prediction of neutrino (1930)

– Discovery of positron (1932)

– Fermi theory (1934)

– Discovery of muon (1937)

– Demonstration that muon interacts too weakly for nuclear forces (1947)

See timeline here: hep-ph/0001283

α-decay and Yukawa theory[1]

- Electromagnetic interactions are mediated by exchange of photons

Yukawa 1935

- Yukawa's idea – proton and neutron exchange **some new massive particle** to interact in the nuclei

  H. Yukawa *On the interaction of elementary particles* Proc.Phys.Math.Soc.Jap. 17 (1935) 48

- Static $p - n$ interaction was known to **decrease rapidly** at the distances $\geq 2\,\mathrm{fm} = 2 \times 10^{-13}\,\mathrm{cm}$.

- The idea of Yukawa – there is $p - n$ interaction of the form

$$U(r) = \frac{g^2}{4\pi r} e^{-r/a} \qquad (1)$$

where $a \lesssim 1\,\mathrm{fm}$

[0]This presentation closely follows Aitchison & Hey, Sec. 2.2 (see also Sec. 2.1 and 2.3–2.5)

α-decay and Yukawa theory

Yukawa interaction	Electromagnetic interaction
$U(r) = \frac{g^2}{4\pi r} e^{-r/a}$ – short ranged	$U(r) = \frac{e^2}{4\pi r}$ – long ranged
Poisson equation	
$\left(\nabla^2 - \frac{1}{a^2}\right) U(r) = g^2 \delta(r)$	$\nabla^2 U(r) = e^2 \delta(r)$
Free-space propagation	
$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \frac{1}{a^2}\right) U(r) = 0$	$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right) U(r) = 0$
De Broglie solution $U \propto \exp\left(\frac{i\boldsymbol{p}\cdot\boldsymbol{x}}{\hbar} - \frac{iEt}{\hbar}\right)$ **implies that**	
$E^2 = p^2c^2 + \frac{c^2\hbar^2}{a^2}$	$E^2 = c^2p^2$
Mass of quanta : $m_Y \equiv \frac{\hbar}{ac}$	Mass of quanta= 0

Massive Klein-Gordon equation

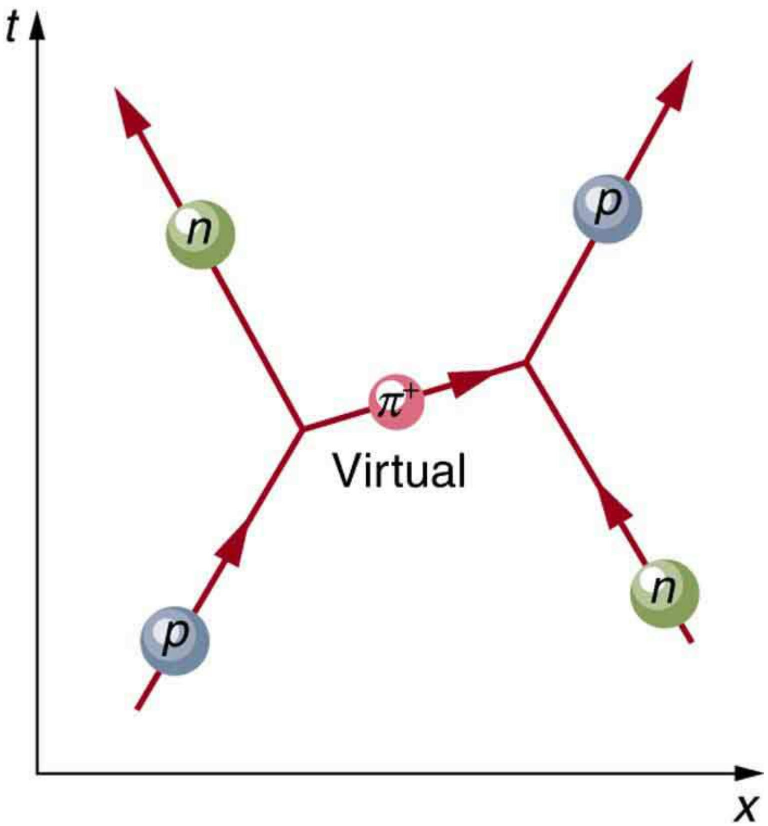
- **Build** the Green's function of the Klein-Gordon equation

$$(\Box + m^2)\phi(x) = \delta^{(4)}(x - x')$$

- use $m^2 \to m^2 - i\epsilon$ to obtain **causal Green's function**
- **Find** its spatial representation. Consider two cases: $(x - x')^\mu = (t, 0)$ and $(x - x')^\mu = (0, \boldsymbol{x})$). Find behavior of the Green's function when $t \to \infty$ or $|\boldsymbol{x}| \to \infty$

Virtual quanta

- Yukawa theory predicted a particle with the mass $m_Y \approx 100\,\mathrm{MeV}/c^2$ (from $a \approx 2\,\mathrm{fm}$), responsible for proton-neutron (nuclear) interactions



- $m_n - m_p \approx 1.3\,\mathrm{MeV}/c^2 \ll m_Y$. Therefore an emission process $n \to p + Y$ is impossible for **real particle** Y (energy conservation violated)

- Exchange by the virtual particle mediates the proton-neutron interaction

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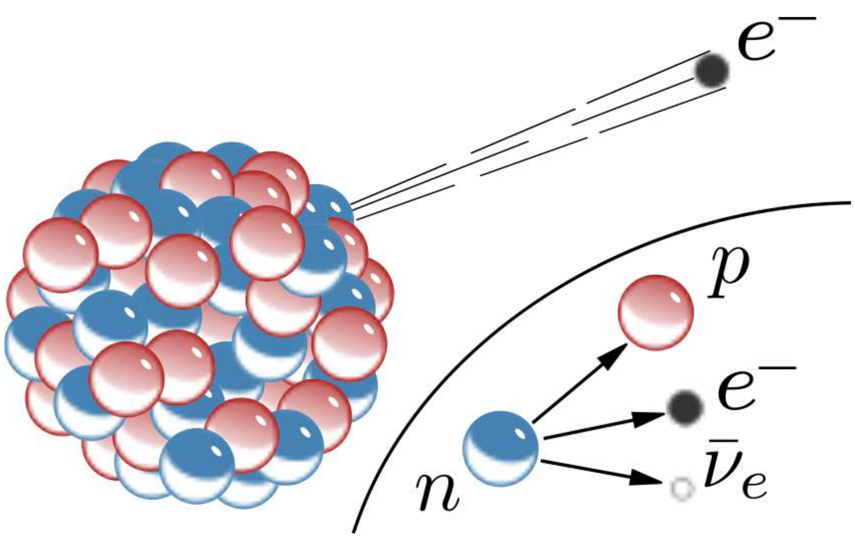
Fermi theory of β-decay [3]



- Neutron decay $n \to p + e^- + \bar{\nu}_e$
- Two papers by E. Fermi:

An attempt of a theory of beta radiation. 1. (In German) Z.Phys. 88 (1934) 161-177
DOI: 10.1007/BF01351864

Trends to a Theory of beta Radiation. (In Italian) Nuovo Cim. 11 (1934) 1-19
DOI: 10.1007/BF02959820

Continuum spectrum of electrons (**1927**)

Prediction of neutrino (**1930, 1934**)

Fermi theory (**1934**)

Universality of Fermi interactions (**1949**)

- Fermi 4-fermion theory:

$$\mathcal{L}_{\text{Fermi}} = -\frac{G_F}{\sqrt{2}}[\bar{p}(x)\gamma_\mu n(x)][\bar{e}(x)\gamma^\mu \nu(x)] \quad (2)$$

- New phenomenological constant, G_F, **Fermi constant.**

[1]History of β-decay (see **[hep-ph/0001283]**, Sec. 1,1); Cheng & Li, Chap. 11, Sec. 11.1)

Fermi theory of β-decay [4]

- Dimension of the Fermi constant? .
  $[\mathcal{L}] = [G_F][(\bar\psi\psi)^2]$. Dimension of Lagrangian – $[\mathcal{L}] = M^4$.[2] Mass operator $M\bar\psi\psi \Rightarrow [\bar\psi\psi] = M^3 \Longrightarrow [G_F] = M^{-2}$

- Value determined experimentally to be $G_F \approx 10^{-5}$ GeV^{-2} (More precisely $\frac{G_F}{(\hbar c)^3} = 1.166 \times 10^{-5}\,\mathrm{GeV}^{-2}$.)

- Fermi Lagrangian includes leptonic and hardronic terms:

$$\mathcal{L}_{\text{Fermi}} = -\frac{G_F}{\sqrt{2}}\left[J^\dagger_{\text{lepton}}(x) + J^\dagger_{\text{hadron}}(x)\right] \cdot \left[J_{\text{lepton}}(x) + J_{\text{hadron}}(x)\right] \quad (3)$$

- Lagrangian (3) predicts universality of weak interactions: all processes in Fermi theory can be described in terms of only one constant, G_F

[2]Dimension $[A_\mu] = [\partial_\mu] = M$. Therefore $[\mathcal{L}] = [F^2_{\mu\nu}] = M^4$.

Neutrino-electron scattering

- Fermi theory predicts **lepton-only** weak interactions, such as $e + \nu_e \to e + \nu_e$ scattering

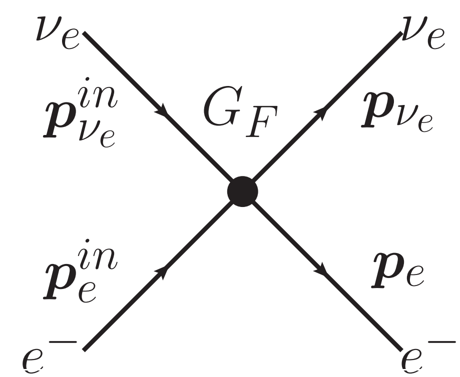
$$\mathcal{L}_{\nu e} = \frac{4G_F}{\sqrt{2}}(\bar{e}\gamma_\lambda \nu_e)(\bar{\nu}_e \gamma^\lambda e) \tag{4}$$

- Matrix element for $e + \nu_e \to e + \nu_e$ scattering $|\mathcal{M}|^2 = \left| \langle e^-, \nu_e | \, \mathcal{L}_{\text{Fermi}} \, | e^-, \nu_e \rangle \right|^2$ (summed over spins of outgoing particles and averaged over spins of initial particles)

$$\begin{aligned} \sum_{\text{spins}} |\mathcal{M}|^2 &\propto \left| G_F \bar{u}(p_e)\gamma_\lambda u(p_e^{in}) \bar{u}(p_\nu)\gamma^\lambda u(p_\nu^{in}) \right|^2 \\ &\propto G_F^2 (p_e^{in} \cdot p_\nu^{in})(p_\nu \cdot p_e) \propto \boxed{G_F^2 E_{c.m.}^4} \end{aligned} \tag{5}$$

Neutrino-electron scattering

- center-of-mass energy of the system is $E^2_{c.m.} = \frac{1}{4}(p_e^{in} + p_\nu^{in})^2 \approx (p_e^{in} \cdot p_\nu^{in})$ for $E_{c.m.} \gg m_e$
- momentum of one of the incoming particles $\boldsymbol{p}_{c.m.} \approx E_{c.m.}$ for $E_{c.m.} \gg m_e$



- Cross-section:

$$d\sigma = \frac{1}{4|\boldsymbol{p}_{c.m.}|E_{c.m.}} \frac{d^3\boldsymbol{p}_e}{(2E_e)} \frac{d^3\boldsymbol{p}_\nu}{(2E_\nu)} |\mathcal{M}|^2 \delta(E_{c.m.} - E_e - E_\nu)\delta^3(\boldsymbol{p}_e + \boldsymbol{p}_\nu) \quad (6)$$

- **Reproduce** this calculation using the Lagrangian (4). Keep in mind that neutrino is a purely massless two-component spinor, i.e.

$$\gamma^\mu p_\mu (1 + \gamma_5)\nu(p) = 0$$

  where $\frac{1}{2}(1 + \gamma_5)$ is the projector that selects only two (left) component of any 4-component spinor
- Show that any solution of the Dirac equation for neutrino depending only on t and one spatial coordinate (z) is “left-moving” (i.e. the wave-function has the general form $\psi(t, z) = f(t + z)$)

Low energy weak processes

- Muon (μ^-) discovered in cosmic rays (charge of the electron, mass $\approx 100\,\mathrm{MeV}$) was first confused with the meson (predicted by Yukawa theory). Originally called μ-meson
- Later π mesons and other mesons were discovered. They were all similar, but μ^- was not interacting with nuclear forces as any meson
- So muon turned out to be just a "heavy electron"
- Fermi Lagrangian (3) was extended so that leptonic current J_{lepton} would include muon:

$$J^\lambda_{\text{lepton}} = \bar\nu_e \gamma^\mu e + \bar\nu_\mu \gamma^\lambda \mu$$

- ...describing for example, the muon decay

$$\mu^- \to e^- + \bar\nu_e + \nu_\mu$$

implies the existence of **muon neutrino**

ν_μ discovered in 1962

- Muon mass $m_\mu \approx 105\,\mathrm{MeV}$. $m_\mu \gg m_e$. In the rest frame of muon e^-, ν_e, ν_μ can be taken as approximately massless.[5]

- The answer should be constructed as a product of two dimensionful quantities. Can construct the only object with dimension of time, proportional to G_F^{-2} .[6]

$$\text{Decay width } \Gamma_\mu \propto \frac{G_F^2 m_\mu^5}{\text{some powers of } \pi \ldots}$$

- **Find** the exact answer: $\Gamma = \frac{G_F^2 m_\mu^5}{192\pi^3}$ using the analog of Lagrangian (4) where one $\bar{e}\gamma_\lambda\nu_e$ term is substituted with $\bar{\mu}\gamma_\lambda\nu_\mu$ term

[5] There can be a caveat here! Will see it when analyzing pion decay

[6] Actually, we will talk about **decay width** with the dimension sec^{-1} or equivalently, GeV. Recall that decay width of $1\,\mathrm{GeV}$ corresponds to the lifetime $6.6 \times 10^{-25}\,\mathrm{sec}$!

Low energy weak processes

- Notice, that similar argument would not work on the neutron's decay:

$$n \to p + e^- + \bar{\nu}_e$$

as this time $m_n \approx m_p$ and therefore one cannot simply write $\Gamma_n \propto$ ~~$G_F^2 m_n^5$~~. Rather it should be some combination of $m_n = 939.5\,\mathrm{MeV}$ and $(m_n - m_p) \approx 1.3\,\mathrm{MeV}$.

Massive intermediate particle

- We saw in Eq. (5) that matrix element for $e + \nu_e \to e + \nu_e$ scattering grows with energy, $|\mathcal{M}|^2 \propto G_F^2 E_{c.m.}^4$.

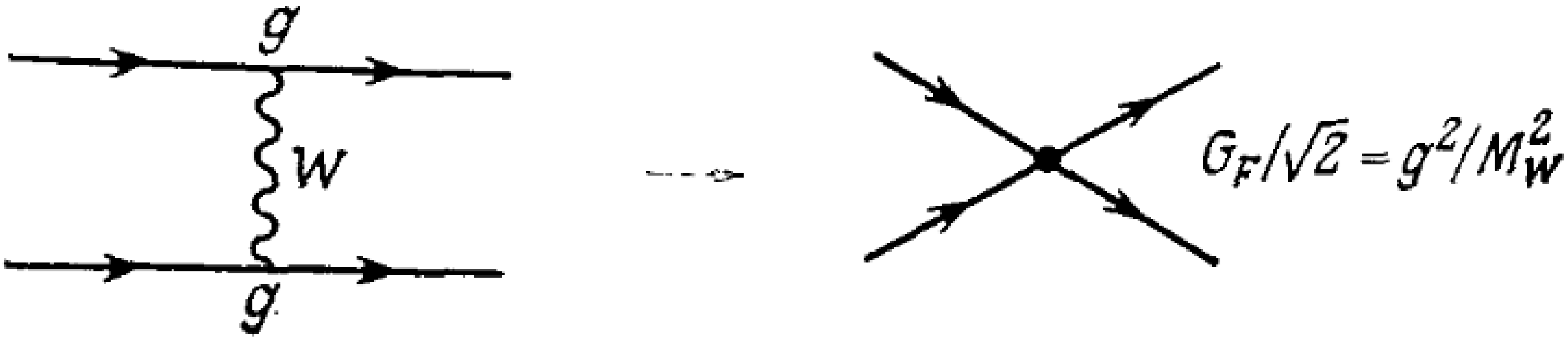
- Unitarity means that this element should be **bounded from above**: $|\mathcal{M}| \le \text{const}$

- Therefore, the Fermi theory would predict meaningless answers for scattering at energies $E_{c.m.} \gtrsim \sqrt{G_F} \approx 300\,\text{GeV}$

- The situation would be ameliorated if G_F were energy dependent and decreasing with energy

- Promote point-like 4-fermion Fermi interaction to interaction, mediated by a **vector boson** : $\mathcal{L}_{\text{Fermi}} \to \mathcal{L}_W = g(W^\mu J_\mu + h.c.)$ 1957

Massive intermediate particle



- Propagator of the massive vector boson:

$$\langle W_\mu(p) W_\nu(-p) \rangle = -i \frac{\eta_{\mu\nu} - \frac{p_\mu p_\nu}{M_W^2}}{p^2 - M_W^2}; \sigma \sim \frac{E^2}{(E^2 - M_W^2)^2}$$

- When mass $M_W \ggg p$ – reduces to Fermi Lagrangian

Vector boson vs. photon

- Propagator of the massive vector boson:

$$\langle W_\mu(p) W_\nu(-p)\rangle = -i\frac{\eta_{\mu\nu} - \frac{p_\mu p_\nu}{M_W^2}}{p^2 - M_W^2 + i\epsilon}$$

- Recall: photon propagator: $\langle A_\mu(p) A_\nu(-p)\rangle = \frac{-i\eta_{\mu\nu}}{p^2+i\epsilon}$

- Try to put $M_W \to 0$. Will you recover photon-like propagator? **No!**
  Trouble with the term in the numerator