Particle Physics of the early Universe

Alexey Boyarsky

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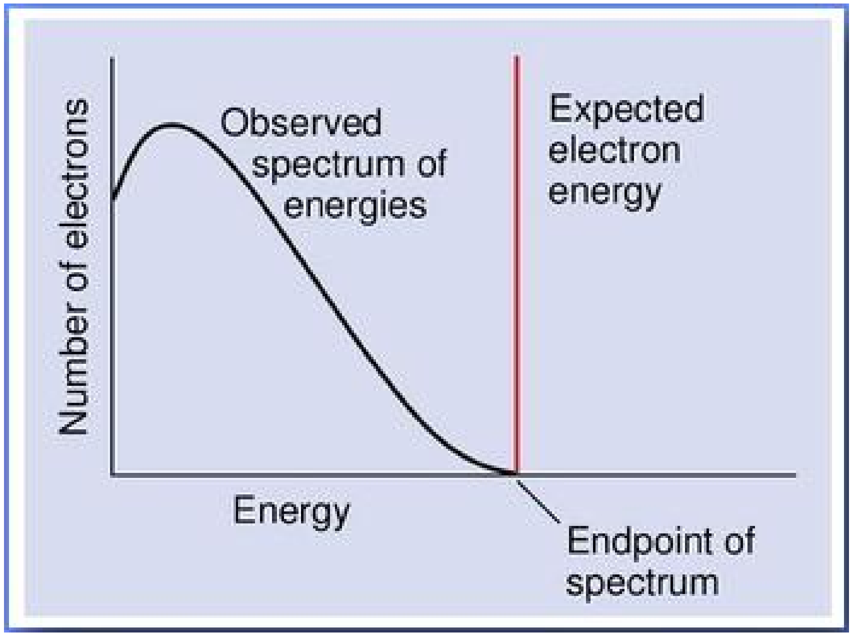
Discovery of neutrino



- Observed $^{14}_{6}C \to ^{14}_{7}N + e^-$
- Two body decay: the electron has the same energy (almost) $\Rightarrow$**not observed!**
- Energy is not conserved?

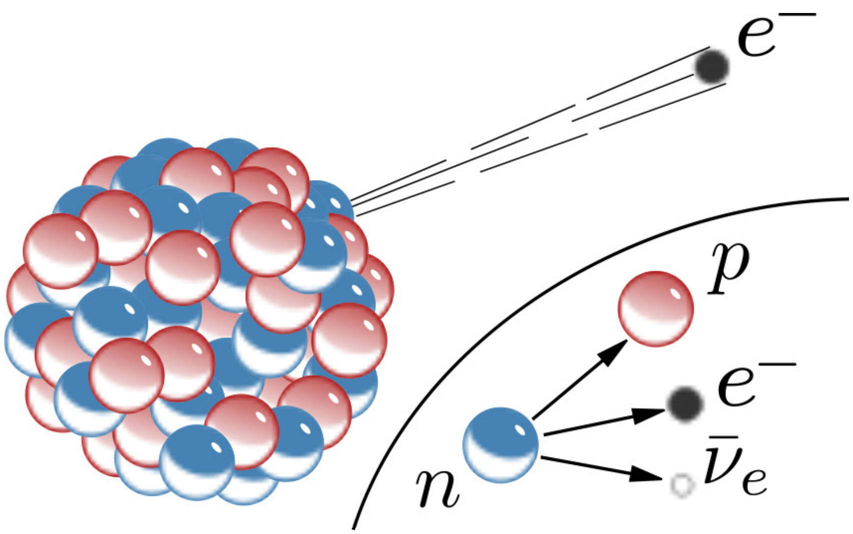
Pauli's letter, Dec. 4, 1930

*...because of the "wrong" statistics of the N and Li^6 nuclei and the continuous beta spectrum, I have hit upon a desperate remedy to save the "exchange theorem" of statistics and the law of conservation of energy. Namely, the **possibility that there could exist in the nuclei electrically neutral particles** $\langle \ldots \rangle$ **which have spin 1/2 and obey the exclusion principle** $\langle \ldots \rangle$. The continuous beta spectrum would then become understandable by the assumption that in beta decay a neutron is emitted in addition to the electron such that the sum of the energies of the neutron and the electron is constant...*

Neutrino

"I have done a terrible thing.
I invented a particle that cannot be detected."
W. Pauli

- Pauli (1930) called this new particle **neutron**
- Chadwick discovered a massive nuclear particle in 1932 $\Rightarrow$**neutron**
- Fermi renamed it into **neutrino** (italian “little neutral one”)



- β-decay is the decay of neutron

$$n \to p + e^- + \bar{\nu}_e$$

inside the nucleus

- Electron capture:

$$^{22}_{11}\text{Na} + e^- \to {}^{22}_{10}\text{Ne} + \nu_e$$

Here ν_e has **opposite spin** than that of $\bar{\nu}_e$!

- The process

$$p + e^- \to n + \nu_e$$

inside a nucleus

β^+-decay

- Also observed was β^+-decay

$$^{22}_{11}\text{Na} \to {}^{22}_{10}\text{Ne} + e^+ + \nu_e$$

Again the particle ν_e has the opposite spin to $\bar{\nu}_e$!

- Formally β^+ decay would come from the

$$p \xrightarrow{?} n + e^+ + \nu_e$$

but mass of proton $m_p < m_n$ (mass of neutron)?!...

- ...possible if neutrons are not free (nuclear binding energy)
- This reaction is the main source of solar neutrinos:

$$4p \to {}^4\text{He} + 2e^+ + 2\nu_e + 26.7\,\text{MeV}$$

Can neutrino be detected?

- Along with $p \xrightarrow{?} n + e^+ + \nu_e$ there can be a reaction $\bar{\nu}_e + p \to n + e^+$ $\Rightarrow$energetic neutrino can cause β^+ decay of a stable nucleus

- In 1934 estimated that the cross-section for such a reaction is tremendously small: $\sigma \sim 10^{-43}\,\mathrm{cm}^2$

  Bethe & Peierls

  for comparison: cross-section of photon scattering on non-relativistic electron (**Thomson cross-section**) is $\sigma_{Thomson} \sim 10^{-24}\,\mathrm{cm}^2$

- In 1942 Fermi had build the first nuclear reactor – source of large number of neutrinos ($\sim 10^{13}\mathrm{neutrinos/sec/cm}^2$)

- Flux of neutrinos from e.g. a nuclear reactor can initiate β^+ decays in protonts of **water**

- Positrons annihilate and two γ-rays and neutron were detected!

Discovered by Cowan & Reines in 1956

How to describe ν_e and $\bar{\nu}_e$

- For massless particles one can write:

$$\begin{aligned} i\sigma^\mu\partial_\mu\nu &= 0 \\ i\bar{\sigma}^\mu\partial_\mu\bar{\nu} &= 0 \end{aligned} \quad (1)$$

(where $\sigma^\mu = (\mathbb{1}, \sigma_1, \sigma_2, \sigma_3)$, $\bar{\sigma}^\mu = (\mathbb{1}, -\sigma_1, -\sigma_2, -\sigma_3)$)

- If neutrinos are **massive** with the mass $m_\nu \lll m_e$

- We know that massive Dirac equation describes evolution of 4-component spinor ψ_ν

$$(i\gamma^\mu\partial_\mu - m_\nu)\psi_\nu = 0$$

- For each $\vec{p}$ there are 4 solutions: two positive frequency ones with positive and negative helicity and two negative frequency ones (or, alternatively, antiparticles) (recall that helicity is a sign of projection of spin onto momentum $\vec{p}$)

- This would predict that there should be particle and anti-particle (neutrino and **anti**-neutrino), each has both helicities

$$\chi = \begin{pmatrix} \textbf{?} \\ \nu \end{pmatrix} \quad ; \quad \bar{\chi} = \begin{pmatrix} \bar{\nu} \\ \textbf{?} \end{pmatrix} \tag{2}$$

$\psi_\nu = (\chi, \bar{\chi})$. Each of χ and $\bar{\chi}$ has spin $\uparrow$ and spin $\downarrow$ components:

$$\psi_\nu \stackrel{?}{=} \begin{pmatrix} \chi(\uparrow) \\ \chi(\downarrow) \\ \bar{\chi}(\uparrow) \\ \bar{\chi}(\downarrow) \end{pmatrix} \tag{3}$$

- So far people detected only neutral particles with spin up or down. **What are these two degrees of freedom?**

How to describe ν_e and $\bar{\nu}_e$

- One possibility: one can put a solution of the Dirac equation, putting $\chi(\downarrow) = \bar{\chi}(\uparrow) \equiv 0$.

  This is impossible for $m_\nu \neq 0$: even if one starts with such a solution, it will change over the course of evolution:

$$\psi_\nu(0) = \begin{pmatrix} 0 \\ \nu \\ 0 \\ 0 \end{pmatrix} \xrightarrow{\text{evolution}} \begin{pmatrix} 0 \\ \nu_1 \\ 0 \\ N_1 \end{pmatrix}$$

- Limiting the evolution to the state $\begin{pmatrix} 0 \\ \nu(x,t) \end{pmatrix}$ can be done if $m_\nu = 0$.

- If this is true, then the prediction is that there are particles ν_e that always have definite spin projection and anti-neutrino with the opposite spin projection.

How to describe ν_e and $\bar{\nu}_e$

- These particles are **distinct** (there is an interaction that differentiates between two)

- Their mass means that there are two more degrees of freedom:

$$\chi \to \begin{pmatrix} \boldsymbol{N} \\ \nu \end{pmatrix} \quad ; \quad \bar{\chi} \to \begin{pmatrix} \bar{\nu} \\ \bar{\boldsymbol{N}} \end{pmatrix} \quad \psi_\nu = \begin{pmatrix} \chi \\ \bar{\chi} \end{pmatrix} \tag{4}$$

- $N, \bar{N}$ do not participate in weak interactions

- **Alternatively**, **identify** $\chi(\uparrow) \leftrightarrow \bar{\chi}(\downarrow)$ and vice versa. Overall, there are only two degrees of freedom:

$$\text{Majorana fermion} \qquad \psi_\nu = \begin{pmatrix} \chi \\ i\sigma_2 \chi^* \end{pmatrix}$$

where the matrix $i\sigma_2 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ interchanges spin-up and spin-down components of the spinor χ^*

How to describe ν_e and $\bar{\nu}_e$

- The corresponding modification of the Dirac equation was suggested by Majorana (1937)

$$i\sigma^\mu \partial_\mu \chi - m_\nu i\sigma_2 \chi^* = 0 \quad (5)$$

Such equation is possible only for **trully neutral particles**

- Indeed, if the particle is not neutral, i.e. if there is a charge under which $\chi \xrightarrow{\hat{Q}} \chi e^{i\alpha Q}$ then χ^* has the opposite charge and equation (5) does not make sense (relates two objects with different transformation properties)

- Notice that when $m_\nu = 0$ there is no difference between Majorana equation for the massless particle and massless 2-component Dirac equation

- In the same year when Cowan & Reines did their experiment, people were not abe to detect the same reactor neutrinos via

$$\bar{\nu}_e + {}^{37}_{17}\text{Cl} \not\longrightarrow \quad {}^{37}_{18}\text{Ar} + e^- \tag{6}$$

- The conclusion was that ν_e (that would be captured in reaction (6)) and $\bar{\nu}_e$ that **was** captured by proton in Cowan & Reines were different particles!

Muon

- Muon, discovered in cosmis rays — heavier “brother” of electron
- Mass: $m_\mu \approx 105$ MeV

Another type of neutrino

- Later it was observed that **muon** decays always via 3-body decay:

$$\mu^- \to e^- + \bar{\nu}_e + \nu_\mu$$

- ... but never via two-body decay, e.g.

$$\mu^- \to e^- + \gamma \quad \leftarrow \textbf{not observed!}$$

- also not observed

$$\mu^- \to e^- + e^+ + e^- \quad \leftarrow \textbf{not observed!}$$

- Pions decay to muons or electrons via two-body decay, emitting some neutral massless particle with the spin 1/2:

$$\pi^+ \to \mu^+ + \nu_\mu \qquad \pi^- \to \mu^- + \bar{\nu}_\mu$$
$$\pi^+ \to e^+ + \nu_e \qquad \pi^- \to e^- + \bar{\nu}_e$$

Detection of other neutrinos

- Muon neutrino ν_μ has been eventually detected via the process:

$$\mu \to e^- + \bar{\nu}_e + \nu_\mu$$
$$\hookrightarrow \nu_\mu + n \to p + \mu^-$$

- If the particle produced in muon decay were ν_e — it would not produce muon in the second reaction (but electron instead)

- Such a reaction was observed in 1962 by Lederman, Schwartz and Steinberger

- Original Fermi theory (1934)

$$\mathcal{L}_{\text{Fermi}} = \frac{G_F}{\sqrt{2}}[\bar{p}(x)\gamma_\mu n(x)][\bar{e}(x)\gamma^\mu \nu(x)] \tag{7}$$

- Universality of weak interactions

before 1957!

$$\mathcal{L}_{\text{Fermi}} = \frac{G_F}{\sqrt{2}} \underbrace{[\bar{p}(x)\gamma_\mu n(x)][\bar{e}(x)\gamma^\mu \nu_e(x)]}_{\beta-\text{decay}} + \frac{G_F}{\sqrt{2}} \underbrace{[\bar{\nu}_\mu(x)\gamma^\mu \mu(x)[\bar{e}(x)\gamma^\mu \nu_e(x)]}_{\text{muon decay}} + \frac{G_F}{\sqrt{2}} \underbrace{[\bar{\nu}_e(x)\gamma^\mu e(x)[\bar{e}(x)\gamma^\mu \nu_e(x)]}_{\text{electron-neutrino scattering}} \dots (\text{pion decay, etc.}) \tag{8}$$

- All processes are governed by the same **Fermi coupling constant:** $G_F \approx 1.16 \times 10^{-5}\,\text{GeV}^{-2}$

Parity and weak interactions

$$\mathcal{L}_{\text{Fermi}} = \frac{G_F}{\sqrt{2}} \underbrace{[\bar{p}(x)\gamma_\mu(1-\gamma_5)n(x)][\bar{e}(x)\gamma^\mu(1-\gamma_5)\nu_e(x)]}_{\beta-\text{decay}} \tag{9}$$

$$+ \frac{G_F}{\sqrt{2}} \underbrace{[\bar{\nu}_\mu(x)\gamma^\mu(1-\gamma_5)\mu(x)[\bar{e}(x)\gamma^\mu(1-\gamma_5)\nu_e(x)]}_{\text{muon decay}}$$

$$+ \frac{G_F}{\sqrt{2}} \underbrace{[\bar{\nu}_e(x)\gamma^\mu(1-\gamma_5)e(x)[\bar{e}(x)\gamma^\mu(1-\gamma_5)\nu_e(x)]}_{\text{electron-neutrino scattering}}$$

$$\ldots (\text{pion decay, etc.})$$

Only left (spin opposed to momentum) neutrinos and right (spin co-aligned with momentum) anti-neutrinos are produced or detected in weak interactions

The weak interactions conserve **flavour lepton numbers**

Tau-lepton

- In (1975), the third lepton, τ, has been discovered. The third type of neutrino, ν_τ as found in (2000)

- To this date there has not been a single detection of $\bar{\nu}_\tau$, although we do believe in its existence

Neutrino in the early Universe

Neutrino properties

- neutrinos are stable
- neutrinos are electrically neutral
- neutrinos participate in weak interactions
- there are 3 neutrinos (for each generation): ν_e, ν_μ, ν_τ
- neutrinos have tiny masses
  (much smaller than the mass of electron, to be discussed later)

How neutrinos are produced in the early Universe?

Neutrino reaction rates?

- Recall: weak interaction strength is **Fermi coupling constant** $G_F \approx 10^{-5}$ GeV^{-2}

- In the processes like $e^+ + e^- \to \nu_\alpha + \bar{\nu}_\alpha$ the interaction rate

$$\Gamma_{ee\to\nu\bar{\nu}} = n_e(T) \times \sigma_{\text{Weak}}$$

where

$$\sigma_{\text{Weak}} \propto G_F^2 \times E_e^2$$

- similarly for electron neutrino reactions $\nu_e + e \to \nu_e + e$ play an important role (we will see below why this is not important for ν_μ, ν_τ)

- What is the typical energy of electrons in this reaction?

- If in the expanding Universe particles that are in thermal equilibrium have either Fermi-Dirac or Bose-Einstein distributions

- At temperatures $T \gg m$ electron distribution function is

$$f_e(p) = 4 \int \frac{d^3p}{(2\pi)^3} \frac{1}{e^{p/T} + 1}$$

- Number density of the electrons + positrons:

$$n_e(T) = g \int \frac{d^3p}{(2\pi)^3} \frac{1}{e^{p/T} + 1} = g \times \frac{3\zeta(3)}{4\pi^2} T^3$$

where $g = 4$ (spin up/down for electron, spin up/down for positron)

- Average energy of the electron $E_e = c \times \langle p \rangle$ i.e

$$E_e = \frac{4}{n_e(T)} \int \frac{d^3p}{(2\pi)^3} \frac{p}{e^{p/T} + 1} \sim T$$

- As a result $E_e \sim T$

- Reaction rate $\Gamma_{ee\to\nu\bar\nu} \sim G_F^2 T^5$

- Compare the characteristic interaction time $\Gamma^{-1}_{ee\to\nu\bar\nu}$ with the age of the Universe $t_{\text{Univ}} = 1/H(T)$. To establish equilibrium we need $\Gamma^{-1}_{ee\to\nu\bar\nu} \ll t_{\text{Univ}}$ or $\Gamma_{ee\to\nu\bar\nu} \gg H(T)$

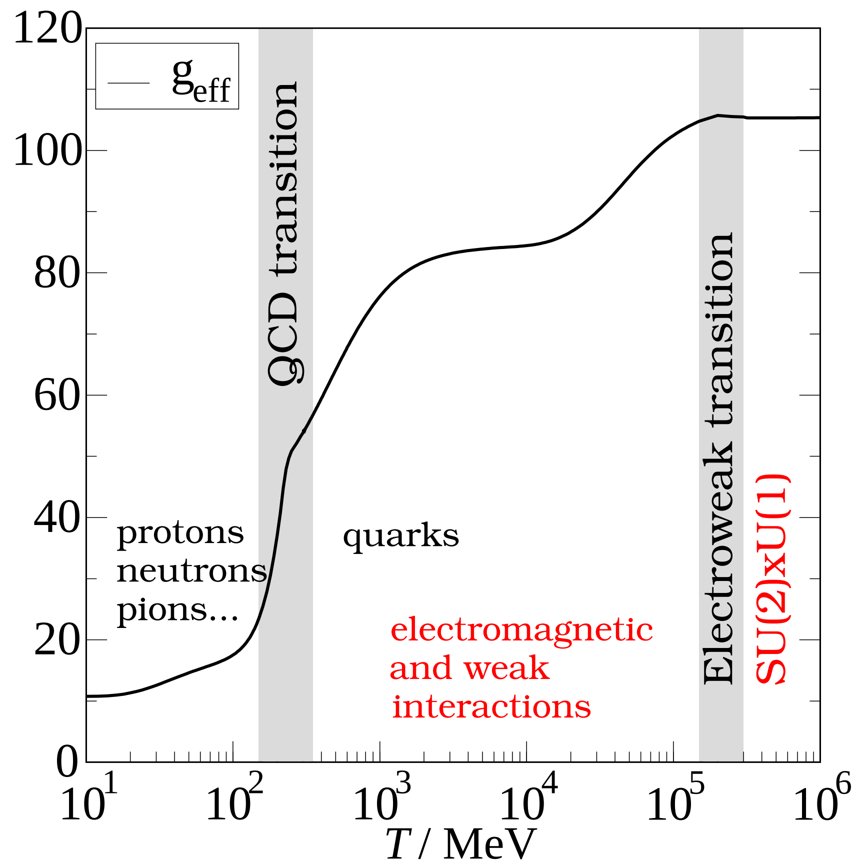
At what temperatures neutrinos are in equilibrium?

- The Friedmanns equation for RD epoch can be written as:

$$H^2(T) = \frac{8\pi G_N}{3} \underbrace{g_*(T)\frac{\pi^2}{30}T^4}_{\rho_{\rm rad}}$$

where $g_* \equiv g_{\rm eff}$ – effective number of relativistic degrees of freedom.

- As a result, $2 \lesssim g_* \lesssim 110$ for Standard Model:



- One can see that temperature when $\Gamma \sim H(T)$ is roughly $T_{\rm dec} \sim 1$ MeV

Neutrino in the early Universe: summary

We saw that

- Neutrinos are produced in the early Universe and are in thermal equilibrium in plasma at $T \gtrsim T_{\text{dec}} \sim 1$ MeV
- As all equilibrium ultra-relativistic particles their average energy is $\langle E_\nu \rangle \sim T$, their number density is $\sim T^3$
- Their interaction rate with other particles $\Gamma_\nu \sim G_F^2 T^5$

What happens below T_{dec}?

Freeze-out

- If $\Gamma \lesssim H$ particle go out of thermal equilibrium – **freeze-out**.
- After the freeze-out, the **comoving number density is conserved** (particles are no longer produced or destroyed):

$$n_{\rm co}(T > T_{\rm dec}) = n_{\rm co}(T_{\rm dec}) \propto T_{\rm dec}^3$$

- The average momentum of decoupled particles changes with time (redshifts). Average momentum at the time of decoupling was ~ 1 MeV. Average momentum today is $\sim 10^{-3}$ eV
- As a result **today** in the Universe there are lots (about $112\ \mathrm{cm}^{-3}$) neutrinos of each flavour (**exercise:** reproduce this number)
- Their **energy density** today:

$$\rho_\nu = \sum m_\nu \times n_{\rm dec} \quad \text{or numerically} \quad \Omega_\nu h^2 \equiv \frac{\rho_\nu}{\rho_{\rm crit}} \approx \frac{\sum m_\nu}{94\ \text{eV}}$$

Neutrino masses and lepton flavour

Fermion number conservation (formally)

- Recall $\mathcal{L} = \bar{\psi}\partial\!\!\!/\psi + m\bar{\psi}\psi$ does not change if $\psi \to \psi e^{i\alpha}$
- Nöther theorem guarantees fermion number conservation:

$$J_F^\mu = \bar{\psi}\gamma^\mu\psi \qquad \partial_\mu J_F^\mu = 0$$

- If there are several **flavours** (ψ_i) then

$$\mathcal{L} = \sum_{i=1}^{N} \bar{\psi}_i\partial\!\!\!/\psi_i + m_i\bar{\psi}_i\psi_i \tag{10}$$

- we have N conserved **fermion (flavour) numbers**

$$J_i^\mu = \bar{\psi}_i\gamma^\mu\psi_i \qquad \partial_\mu J_i^\mu = 0$$

As a consequence $J_F^\mu = \sum_i J_i^\mu$ is also conserved

Flavour lepton numbers

- Define flavor lepton numbers L_e, L_ν, L_τ:

	L_e	L_μ	L_τ			L_e	L_μ	L_τ
(ν_e, e^-)	+1	0	0		$(\bar\nu_e, e^+)$	-1	0	0
(ν_μ, μ^-)	0	+1	0		$(\bar\nu_\mu, \mu^+)$	0	-1	0
(ν_τ, τ^-)	0	0	+1		$(\bar\nu_\tau, \tau^+)$	0	0	-1

- Total lepton number is $L_{tot} = L_e + L_\mu + L_\tau$.

- **Symmetry of the Standard Model:** conserved **flavour lepton number** and **total lepton number**

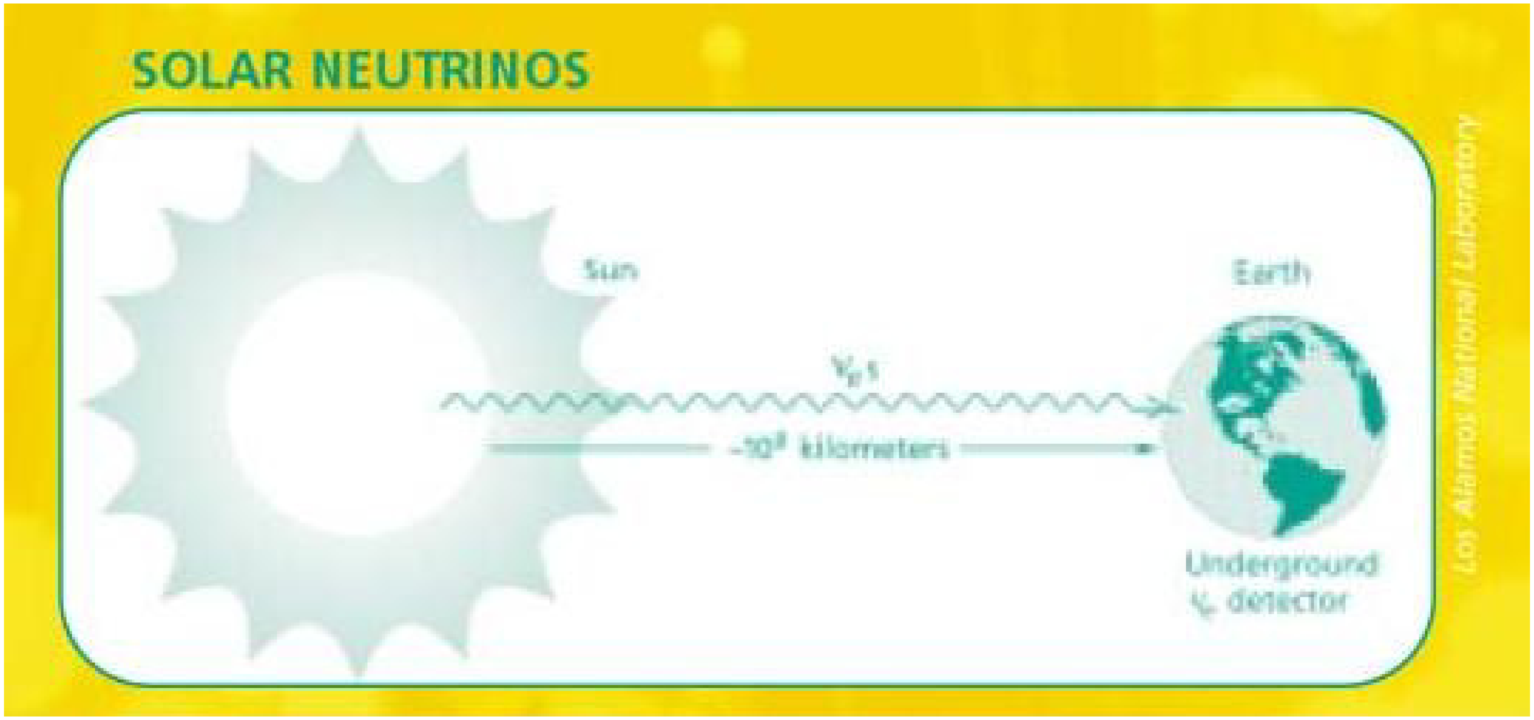
- Fermi interactions respect this symmetry

$$\mathcal{L} = \begin{pmatrix} \bar\nu_e \\ \bar\nu_\mu \end{pmatrix} \underbrace{\left[i\partial\!\!\!/ \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} V^e_{\text{Fermi}} & 0 \\ 0 & V^\mu_{\text{Fermi}} \end{pmatrix} \right]}_{\text{weak interactions}} \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix}$$

Neutrino experiments

- ⋆ The **atmospheric** evidence: disappearance of ν_μ and $\bar{\nu}_\mu$ atmospheric neutrinos ($\nu_\mu \to \nu_\tau$) SuperKamioka
- ⋆ The **solar** evidence: deficit $\sim 50\%$ of solar ν_e ($\nu_e \to \nu_{\mu,\tau}$) SNO
- ⋆ The **reactor** evidence: disappearance of $\bar{\nu}_e$ produced by nuclear reactors.

Back to neutrinos KamLAND



How is this possible if weak interactions conserve flavour?

Mass and charge eigenstates

- Define **charge eigenstates** as those where interaction term is diagonal

$$\mathcal{L} = \begin{pmatrix}\bar\nu_e \\ \bar\nu_\mu\end{pmatrix} \underbrace{\left[i\partial\!\!\!/ \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} + \begin{pmatrix}V^e_{\text{Fermi}} & 0\\ 0 & V^\mu_{\text{Fermi}}\end{pmatrix}\right]}_{\text{weak interactions}} \begin{pmatrix}\nu_e \\ \nu_\mu\end{pmatrix}$$

- For exampe, $V_{\text{Fermi}} \sim G_F n_e$ if neutrinos propagate in the medium with high density of electrons (interior of the Sun)

- If neutrinos have mass, the mass term is not necessarily diagonal in this basis:

$$\mathcal{L} = \begin{pmatrix}\bar\nu_e \\ \bar\nu_\mu\end{pmatrix} \underbrace{\left[i\partial\!\!\!/ \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} + \begin{pmatrix}V^e_{\text{Fermi}} & 0\\ 0 & V^\mu_{\text{Fermi}}\end{pmatrix}\right]}_{\text{weak interactions}} \begin{pmatrix}\nu_e \\ \nu_\mu\end{pmatrix} + \begin{pmatrix}\bar\nu_e \\ \bar\nu_\mu\end{pmatrix} \begin{pmatrix}m_{11} & m_{12}\\ m_{21} & m_{22}\end{pmatrix} \begin{pmatrix}\nu_e \\ \nu_\mu\end{pmatrix}$$

Mass and charge eigenstates

- One can define **mass eigenstates** such that the kinetic plus mass term is diagonal in this basis

$$\mathcal{L} = \begin{pmatrix} \bar{\psi}_1 \\ \bar{\psi}_2 \end{pmatrix} \left[i\not{\partial} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix} \right] \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} + \begin{pmatrix} \bar{\psi}_1 \\ \bar{\psi}_2 \end{pmatrix} \begin{pmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{pmatrix} \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$$

- A unitary transformation rotates between these two choices of basis

$$\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \underbrace{\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}}_{\text{matrix } U} \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$$

Neutrino oscillations

- Consider the simplest case: two flavours, two mass eigen-states. Matrix U is parametrized by one **mixing angle** θ

$$\begin{aligned} |\nu_e\rangle &= \cos\theta\,|1\rangle + \sin\theta|2\rangle \\ |\nu_\mu\rangle &= \cos\theta|2\rangle - \sin\theta|1\rangle \end{aligned}$$

- Let take the initial state to be ν_e (created via some weak process) at time $t = 0$:

$$|\psi_0\rangle = |\nu_e\rangle = \cos\theta\,|1\rangle + \sin\theta\,|2\rangle$$

- Then at time $t > 0$

$$|\psi_t\rangle = e^{-iE_1 t}\cos\theta\,|1\rangle + \sin\theta\,|2\rangle\, e^{-iE_2 t}$$

- We detect the particle later via another weak process (e.g. $\nu_? + n \to p + \mu^-/e^-$)

- The probability of conversion $\nu_e \to \nu_\mu$ is given by

$$P(\nu_e \to \nu_\mu) = |\langle \nu_\mu | \psi_t \rangle|^2 = \sin^2(2\theta) \sin^2\left(\frac{(E_2 - E_1)t}{2}\right)$$

- The probability to detect ν_e is give by

$$P(\nu_e \to \nu_e) = |\langle \nu_e | \psi_t \rangle|^2 = 1 - \sin^2(2\theta) \sin^2\left(\frac{(E_2 - E_1)t}{2}\right)$$

Fermion number conservation?

- Apparent violation of flavour lepton number for neutrinos **can be explained** by the presene of the non-zero neutrino mass

$$\mathcal{L} = \begin{pmatrix} \bar\nu_e \\ \bar\nu_\mu \\ \bar\nu_\tau \end{pmatrix} \underbrace{\left[i\not\partial - V_{\text{Fermi}} \right]}_{\text{conserves flavour number}} \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} + \begin{pmatrix} \bar\nu_e \\ \bar\nu_\mu \\ \bar\nu_\tau \end{pmatrix} \begin{pmatrix} m_{11} & m_{12} & \dots \\ m_{21} & m_{22} & \dots \\ \dots & \dots & \dots \end{pmatrix} \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}$$

- In this case only **one** fermion current (total lepton fermion number) is conserved:

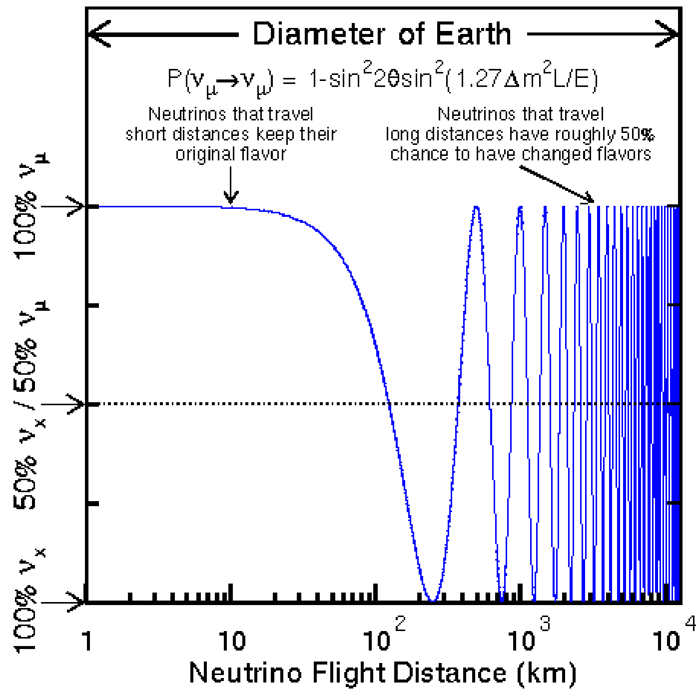
$$J^\mu = \sum_{i=e,\mu,\tau} \bar\nu_i \gamma^\mu \nu_i \tag{11}$$

  while any independent $J_i^\mu = \bar\nu_i \gamma^\mu \nu_i$ is not conserved.

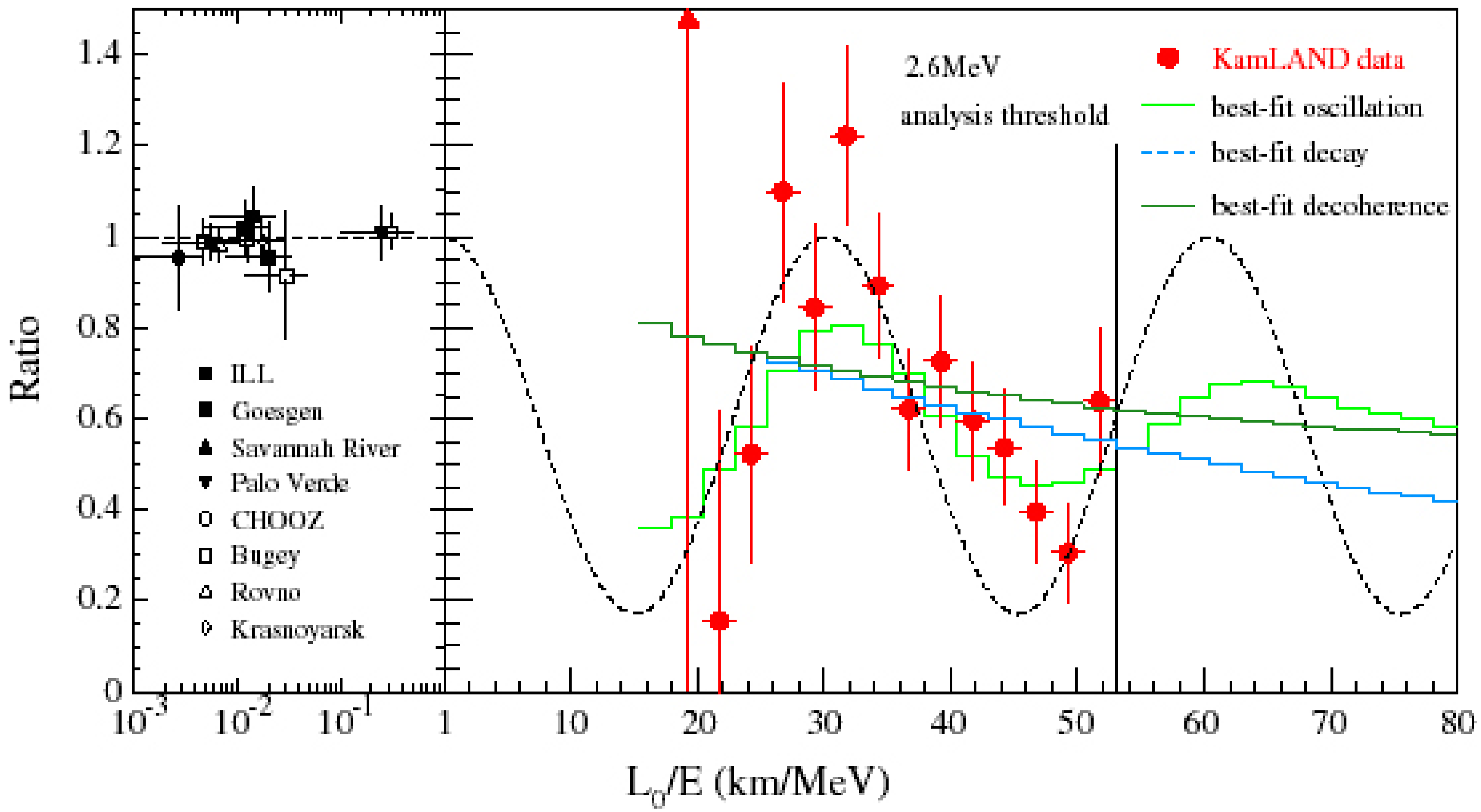
- The prediction is: neutrinos **oscillate**, i.e. probability to observe a given flavour changes with the distances travelled:

$$P_{\alpha\to\beta} = \sin^2(2\theta)\,\sin^2\left(1.267\frac{\Delta m^2 L}{E}\frac{\mathrm{GeV}}{\mathrm{eV}^2\,\mathrm{km}}\right) \quad (12)$$

Mass difference: Δm^2, Neutrino energy: E (keV-MeV in stars, GeV in air showers, etc. Distance traveled: L



- Eq. (12) predicts that probability oscillates as a function of the ratio E/L. This is indeed observed:



in this plot the distance between reactor and detector and energy is different, therefore E/L is different

- Neutrino experiments determine **two** mass splittings between **three** mass eigenstates (m_1, m_2, m_3):

$$\Delta m^2_{\text{solar}} = 7.6\times 10^{-5}\,\text{eV}^2$$
$$|\Delta m^2_{\text{atm}}| = 2.4\times 10^{-3}\,\text{eV}^2$$

- The experiment LSND have claimed evidence for $\bar\nu_\mu \to \bar\nu_e$ oscillations with $\Delta m^2 \sim 1\,\text{eV}^2$ (produced $\bar\nu_\mu$, observed an excess of $\bar\nu_e$)

- MiniBooNE did not confirm a similar scale oscillations of $\nu_\mu \to \nu_e$. Later MiniBooNE found some ν_e excess

- The $\Delta m^2 \sim 1\,\text{eV}^2$ anomaly has never been confirmed at statistically significant level. The data is always in “mild disagreement” with one another. If the signal is real, it is difficult to reconcile all the neutrino oscillation experiments (and even more difficult to bring them in accordance with cosmology)

3 neutrino generations

- A 3×3 unitary transformation U relates mass eigenstates (ν_1, ν_2, ν_3) to flavour eigenstates

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

- Any unitary 3×3 martix has 9 real parameters:

$$U = Exponent \left[i \begin{pmatrix} \lambda_1 & |u_{12}|e^{i\delta_{12}} & |u_{13}|e^{i\delta_{13}} \\ |u_{12}|e^{-i\delta_{12}} & \lambda_2 & |u_{23}|e^{i\delta_{23}} \\ |u_{13}|e^{-i\delta_{13}} & |u_{23}|e^{-i\delta_{23}} & \lambda_3 \end{pmatrix} \right]$$

How many of them can be measured in experiments?

Neutrino mixing matrix

- **Recall** that neutrinos $\nu_{e,\mu,\tau}$ couple to charged leptons $\Rightarrow$ Invariant under $\nu_e \to \nu_e e^{i\alpha}$ simultaneously with $e^- \to e^- e^{i\alpha}$, etc.

- All other terms in the Lagrangian have the form $\bar{\psi}\not{D}\psi$ or $m\bar{\psi}\psi$ — i.e. are invariant if $\psi \to \psi e^{i\alpha}$ (here ψ is any of $\nu_e, \nu_\mu, \nu_\tau, e, \mu, \tau$)

- Additionally, we can rotate each of the $\nu_{1,2,3}$ by an independent phase

- 5 of 9 parameters of the mixing matrix U can be absorbed in the redefinitions of $\nu_{1,2,3}$ and $\nu_{e,\mu,\tau}$ (6th phase does is overall redefinition of all fields – does not change U).

Neutrino mixing matrix

- The rest $9 - 5 = 4$ parameters are usually chosen as follows: **3 mixing angles** $\theta_{12}, \theta_{23}, \theta_{13}$ and **1 phase** ϕ (since 3×3 real orthogonal matrix has 3 parameters only)

Three rotations plus **one** phase ϕ:

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_{23} & \sin\theta_{23} \\ 0 & -\sin\theta_{23} & \cos\theta_{23} \end{pmatrix} \begin{pmatrix} \cos\theta_{13} & 0 & e^{-i\phi}\sin\theta_{13} \\ 0 & 1 & 0 \\ -e^{i\phi}\sin\theta_{13} & 0 & \cos\theta_{13} \end{pmatrix} \begin{pmatrix} \cos\theta_{12} & \sin\theta_{12} & 0 \\ -\sin\theta_{12} & \cos\theta_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Problems for mixing matrix U

1. Show that for two flavour and two mass eigenstates the matrix U has **1** real free parameter (a **mixing angle**)

2. Show that if any of the angle θ_{12}, θ_{23} or θ_{13} is equal to zero, the matrix U can be chosen real

See-saw Lagrangian

Add right-handed neutrinos N_I to the Standard Model

$$\mathcal{L}_{\text{right}} = i\bar{N}_I \partial\!\!\!/ N_I + \begin{pmatrix} \bar{\nu}_e \\ \bar{\nu}_\mu \\ \bar{\nu}_\tau \end{pmatrix} \underbrace{\begin{pmatrix} F\langle H \rangle \end{pmatrix}}_{\text{Dirac mass } M_D} \begin{pmatrix} N_1 \\ N_2 \\ \dots \end{pmatrix} + \begin{pmatrix} N_1^c \\ N_2^c \\ \dots \end{pmatrix} \underbrace{\begin{pmatrix} M \end{pmatrix}}_{\text{Majorana mass}} \begin{pmatrix} N_1 \\ N_2 \\ \dots \end{pmatrix}$$

$\nu_\alpha = \tilde{H} L_\alpha$, where L_α are left-handed lepton doublets

- Active masses are given via usual **see-saw formula**:

$$(m_\nu) = -m_D \frac{1}{M_I} m_D^T \qquad ; \qquad m_D \ll M_I$$

- Neutrino mass matrix – **7 parameters**. Dirac+Majorana mass matrix – **11 (18) parameters** for 2 (3) sterile neutrinos. **Two** sterile neutrinos are enough to fit the neutrino oscillations data.

Problems about see-saw Lagrangian

1. Demonstrate that knowing the masses of all neutrinos does not allow to fix the scale of masses m_D and M_M.

2. Consider the Lagrangian with only one flavour and introduce one singlet right handed neutrino ν_R, and add both Majorana mass term to it and Dirac mass term via Higgs mechanism

$$\mathcal{L}_{\text{seesaw}} = \mathcal{L}_{SM} + \lambda_N \bar{L}_e H^c (\nu_R) + \frac{1}{2} M_M (\bar{\nu}_R^c)(\nu_R) + \text{h.c.}\,. \qquad (13)$$

   Suppose, that the Dirac mass $m_D = \lambda_N v$ is much smaller than Majorana mass M_M, $m_D \ll M_M$. Find the spectrum (mass eigenstates) in (13). Identify linear combinations of ν and N that are mass **mass eigenstates** and rewrite the Lagrangian in this basis. Do not forget that the left double L_e participates in electric and weak interactions.

3. Generalize the above see-saw Lagrangian (13) for the number of SM lepton flavors other than one.

Problems about see-saw Lagrangian

4. Can one obtain the observed mass splittings (see e.g. PDG) by adding only one right-handed neutrino in the three-flavour generalization of the Lagrangian (13)?

5. Generalize the above see-saw Lagrangian (13) for all three SM lepton flavors **and** $\mathcal{N}$ generations of right-handed neutrinos. How many new parameters appears in the see-saw Lagrangian for the case of $\mathcal{N} = 1, 2, 3$?