Particle Physics of the early Universe

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Why do you need that?

- Recall the Dirac Hamiltonian

$$i\hbar\frac{\partial\psi}{\partial t} = \underbrace{\left[-i\hbar c\left(\alpha_x\frac{\partial}{\partial x} + \alpha_y\frac{\partial}{\partial y} + \alpha_z\frac{\partial}{\partial z}\right) + \beta mc^2\right]}_{\text{Dirac Hamiltonian}}\psi \tag{1}$$

- where

$$\alpha_i = \begin{pmatrix} 0 & \sigma_i \\ \sigma_i & 0 \end{pmatrix}; \qquad \beta = \begin{pmatrix} \mathbb{1} & 0 \\ 0 & -\mathbb{1} \end{pmatrix} \tag{2}$$

- The Dirac equation

$$\left(i\frac{\partial}{\partial x^\mu}(\gamma^\mu)_{\alpha\beta} - m\,\delta_{\alpha\beta}\right)\psi^\beta = 0 \tag{3}$$

is the Euler-Lagrange equation of the following action:

$$S_{\text{Dirac}}[\psi,\bar\psi] = \int d^4x\,\bar\psi^\alpha\left(i\frac{\partial}{\partial x^\mu}(\gamma^\mu)_{\alpha\beta} - m\,\delta_{\alpha\beta}\right)\psi^\beta \tag{4}$$

where $\bar{\psi} \equiv \psi^\dagger \gamma_0$ – independent spinor

involves 4 Dirac matrices γ^μ (index $\mu = 0, 1, 2, 3$), each matrix having size 4×4 (indices α, β run over their dimensions)

$$\gamma^0 = \begin{pmatrix} \mathbb{1} & 0 \\ 0 & -\mathbb{1} \end{pmatrix}; \qquad \gamma^i = \begin{pmatrix} 0 & \sigma_i \\ -\sigma_i & 0 \end{pmatrix} \tag{5}$$

- The "plane wave solution" of the Dirac equation

$$\psi(x) = u(p) e^{-ipx} \tag{6}$$

where 4-component complex **spinor** $u(p)$ obeys the equation

$$(p_\mu \gamma^\mu - m) u(p) = 0 \tag{7}$$

- In the same paper [Proc. R. Soc. Lond. (1928) 610, 24] Dirac introduced coupling of spinors to electromagnetic field

- Recall in non-relativistic quantum mechanics coupling to the electromagnetic field was via "minimal coupling" (i.e. the momentum $\hat{p}_\mu \to \hat{P}_\mu = (\hat{p}_\mu - eA_\mu)$, P_μ is sometimes called "generalized momentum")

$$\frac{\hat{p}^2}{2m} \to \frac{1}{2m}\left(\hat{p} - e\vec{A}\right)^2 + eA_0(x) \tag{8}$$

- In Dirac equation, we make similar substitution

$$(\hat{p}_\mu \gamma^\mu - eA_\mu \gamma^\mu - m)\psi = 0 \tag{9}$$

- Dirac Hamiltonian in the electromagnetic field

$$\mathcal{H} = \mathcal{H}_0 + e(A_0 + \gamma_0 \vec{\gamma} \cdot \vec{A}) \tag{10}$$

[2]Bjorken-Drell, Chap. 1, Sec. 1.4

Probability and current density

- Dirac has constructed an equation of the form $\boxed{(i\partial_t - \mathcal{H})\psi = 0}$ where $\mathcal{H}$ is Hermitian

- Therefore the quantity
$$\int d^3x\, \psi^+\psi \tag{11}$$
is conserved

- The quantity
$$\psi^\dagger\psi = |\psi_1|^2 + |\psi_2|^2 + |\psi_3|^2 + |\psi_4|^2 \geq 0 \tag{12}$$
can be interpreted as the **probability density**. Contrary to the Klein-Gordon case, it is non-negative by construction.

- The density $\psi^\dagger\psi$ is a part of the current vector
$$j^\mu = \psi^\dagger\gamma^0\gamma^\mu\psi \tag{13}$$

- As a consequence of the Dirac equation, this current is conserved[4]

$$\partial_\mu j^\mu = 0 \tag{14}$$

therefore the spatial part of j^μ has the meaning of the current density.

[4] Check this

Negative energy states

- As for any relativistic equation, Dirac equation has two branches of dispersion relation

$$E = \sqrt{\boldsymbol{p}^2 + m^2} \quad \text{and} \quad E = -\sqrt{\boldsymbol{p}^2 + m^2} \tag{15}$$

- Consider a state with negative energy ($p_\mu = (-|E|, \boldsymbol{p})$):

$$\begin{aligned} \psi(x) = u(p)e^{-ip\cdot x} = u(p)e^{+i|E|t+i\boldsymbol{p}\cdot\boldsymbol{x}} \\ (p_\mu\gamma^\mu - m)u(p) = 0 \end{aligned} \tag{16}$$

  Naively, if the system contains negative energy states – it is unstable, since it will try to choose the state with the lowest possible energy, but there is no natural lower bound on the value of the negative energy.

- What should be there interpretation?

Negative energy states

- Observe that complex conjugated spinor $\psi^* = u^* e^{+ip\cdot x}$ has **positive** energy:

$$\psi^* = u^* e^{-i|E|t - i\boldsymbol{p}\cdot\boldsymbol{x}} \tag{17}$$

> **Idea:** build a positive energy state, ψ_c, corresponding to the negative energy solution (16) and satisfying the Dirac equation whenever ψ does.

Guess 1: Does the spinor (17) obey the Dirac equation?

$$(i\partial\!\!\!/ - m)\psi^* = e^{+ip\cdot x}(-p_\mu\gamma^\mu - m)u^* \stackrel{?}{=} 0 \tag{18}$$

Notice that as a consequence of Eq. (16) we have

$$\Big(p_\mu(\gamma^\mu)^* - m\Big)u^*(p) = 0 \tag{19}$$

This **could work** (as a consequence of (19)) if gamma-matrices were imaginary:

$$(\gamma^\mu)^* \stackrel{?}{=} -\gamma^\mu \tag{20}$$

This is **not** the case. In the representation that we use (Eq. (5))

– $\gamma^0, \gamma^1, \gamma^3$ are real

– γ^2 is imaginary (show it)

Guess 2: In addition to complex conjugation rotate the spinor $u(p)$:

$$\psi_c = C\psi^* = (Cu^*)e^{+ip\cdot x} \tag{21}$$

$$(i\partial\!\!\!/ - m)\psi_c = e^{+ip\cdot x}(-p_\mu\gamma^\mu - m)Cu^* \stackrel{?}{=} 0 \tag{22}$$

where matrix C is chosen in such a way that

$$-\gamma^\mu C = C(\gamma^\mu)^* \tag{23}$$

(and also $C^2 = 1$). Exercise: demonstrate that this matrix is given by

$$C = i\gamma^2 \tag{24}$$

(trivial consequence of $\{\gamma_\mu, \gamma_\nu\} = \eta_{\mu\nu}$ and γ^2 being imaginary).

- Now consider Dirac equation in the external electromagnetic field:

$$\Big(i\partial\!\!\!/ - e A\!\!\!/ - m\Big)\psi = 0 \tag{25}$$

The spinor $\psi_c = C\psi^*$ obeys the equation:

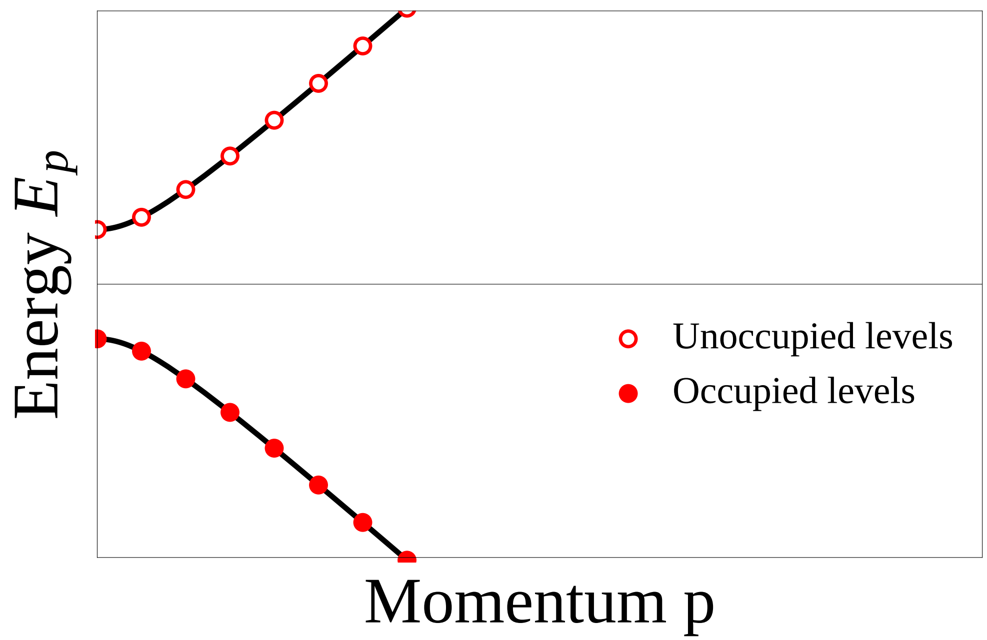
$$\Big(i\partial\!\!\!/ + e A\!\!\!/ - m\Big)\psi_c = 0 \tag{26}$$

Spinor ψ_c describes a state with the opposite charge

Dirac sea

- The interpretation of Dirac: Since fermions obey the Pauli principle, the occupation number of each energy level cannot exceed 1[5]

- Assume that we start from the many-fermion system, where all the energy states with $E < 0$ are fully occupied (the **Dirac sea**).



- This fully-occupied state is interpreted as the **vacuum**

- Such vacuum is stable due to Pauli exclusion princple

[5] Actually it cannot exceed 2, when we take into account two possible spin states

Dirac sea

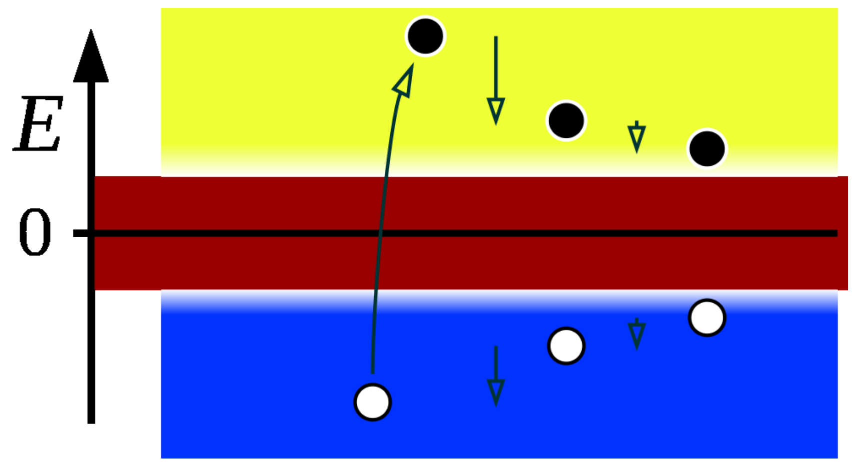
- When we add an electron to the vacuum, it increases overall energy by $E > 0$. The dynamics of this additional particle is described by the positive-energy solutions of the Dirac equation.

- We may also **remove** one particle from the vacuum. The resulting state is called "hole", and behaves like the absence of electron with $E = -\sqrt{\boldsymbol{p}^2 + m^2}$. The removal of fermion with negative energy **increases** the energy of the system. The vacuum has the lowest energy

Holes

Therefore, the hole (i.e. the spinor $\psi_c = C\psi^*$ where $\psi = ue^{-ip\cdot x}$ with the negative energy) has[6]

- **positive** charge $e_{\text{hole}} = +|e|$
- **positive** energy $E_{hole} = +\sqrt{\boldsymbol{p}^2 + m^2}$
- opposite momentum $\boldsymbol{p}_{\text{hole}} = -\boldsymbol{p}$



[6]The analogy is an air bubble in water, compared to the drop of water in air: effectively, the bubble behaves like a drop with negative density.

Prediction of antiparticles

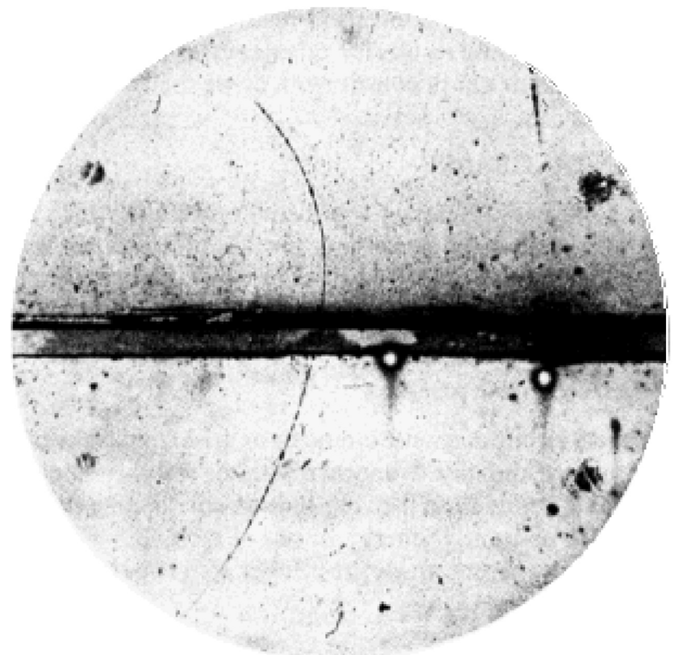
- The hole state is called the **antiparticle**
- For electron, there should exist a positively-charged antiparticle – **positron**. It was first predicted by Dirac.
- In 1932, positron was discovered by Anderson in cosmic rays
  Phys. Rev. 43, 491494 (1933) "*The positive electron*"
  `http://link.aps.org/doi/10.1103/PhysRev.43.491`

Prediction of antiparticles

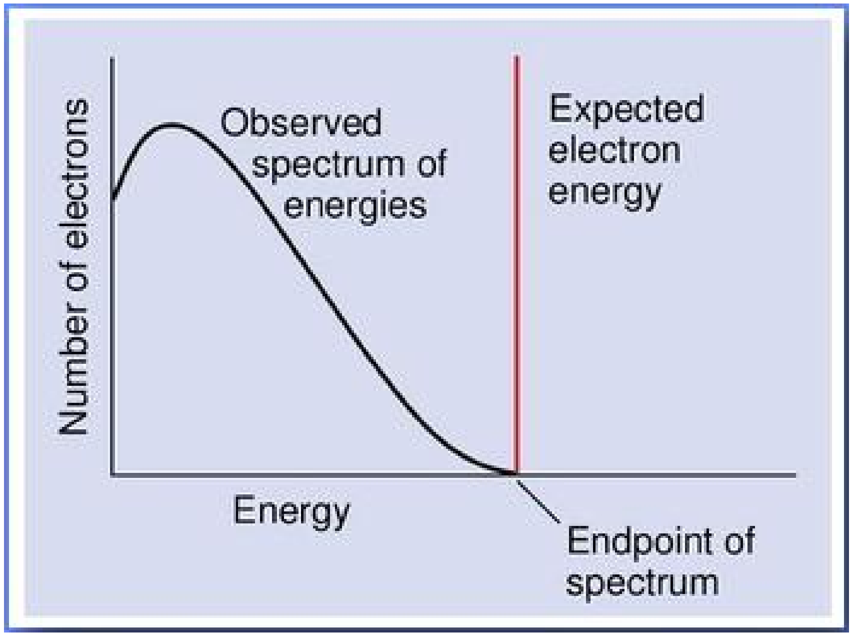
From **Phys. Rev. 43, 491494 (1933) "*The positive electron*"**

http://link.aps.org/doi/10.1103/PhysRev.43.491

Abstract. Out of a group of 1300 photographs of cosmic-ray tracks in a vertical Wilson chamber 15 tracks were of positive particles which could not have a mass as great as that of the proton. From an examination of the energy-loss and ionization produced it is concluded that the charge is less than twice, and is probably exactly equal to, that of the proton. If these particles carry unit positive charge the curvatures and ionizations produced require the mass to be less than twenty times the electron mass. These particles will be called positrons. Because they occur in groups associated with other tracks it is concluded that they must be secondary particles ejected from atomic nuclei.



Quantum mechanics of weak interactions



- Observed ${}^{14}_{6}C \to {}^{14}_{7}N + e^-$
- $Q = M_{A,Z}c^2 - M_{A,Z+1}c^2 > 0$
- Two body decay: the electron has the same energy (almost) $\Rightarrow$**not observed!**
- Energy is not conserved?

Pauli's letter, Dec. 4, 1930

. . . because of the "wrong" statistics of the N and Li[6] nuclei and the continuous beta spectrum, I have hit upon a desperate remedy to save the "exchange theorem" of statistics and the law of conservation of energy. Namely, the ***possibility that there could exist in the nuclei electrically neutral particles*** $\langle \ldots \rangle$ ***which have spin 1/2 and obey the exclusion principle*** $\langle \ldots \rangle$. *The continuous beta spectrum would then become understandable by the assumption that in beta decay a neutron is emitted in addition to the electron such that the sum of the energies of the neutron and the electron is constant. . .*

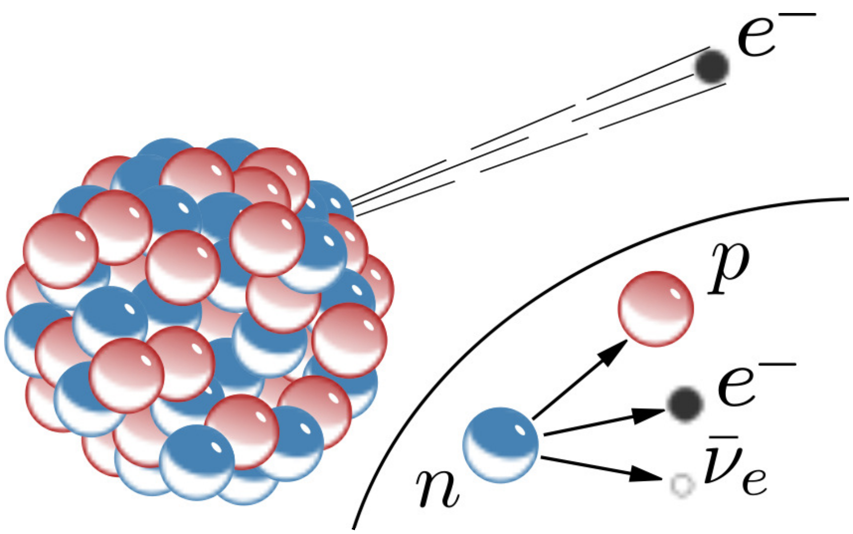
[6]History of β-decay (see **[hep-ph/0001283]**, Sec. 1,1); Cheng & Li, Chap. 11, Sec. 11.1)

Neutrino

"I have done a terrible thing.
I invented a particle that cannot be detected."
W. Pauli

- Pauli (1930) called this new particle **neutron**
- Chadwick discovered a massive nuclear particle in 1932 that was called **neutron**. Its mass was almost equal to the mass of proton $\Rightarrow$not a particle, predicted by Pauli
- Fermi renamed Pauli's particle into **neutrino** (italian "little neutral one")

Neutrino



- Fermi: β-decay is the decay of neutron

$$n \to p + e^- + \bar{\nu}_e$$

inside the nucleus

- Two papers by E. Fermi:

An attempt of a theory of beta radiation. 1. (In German) Z.Phys. 88 (1934) 161-177
DOI: 10.1007/BF01351864

Trends to a Theory of beta Radiation. (In Italian) Nuovo Cim. 11 (1934) 1-19
DOI: 10.1007/BF02959820

- Electron capture:

$$^{22}_{11}\text{Na} + e^- \to {}^{22}_{10}\text{Ne} + \nu_e$$

Here ν_e has **opposite spin** than that of $\bar{\nu}_e$!

- The process $p + e^- \to n + \nu_e$ inside a nucleus

β^+-decay

- Also observed was β^+-decay

$$^{22}_{11}\text{Na} \to {}^{22}_{10}\text{Ne} + e^+ + \nu_e$$

Again the particle ν_e has the opposite spin to $\bar{\nu}_e$!

- Formally β^+ decay would come from the

$$p \overset{?}{\to} n + e^+ + \nu_e$$

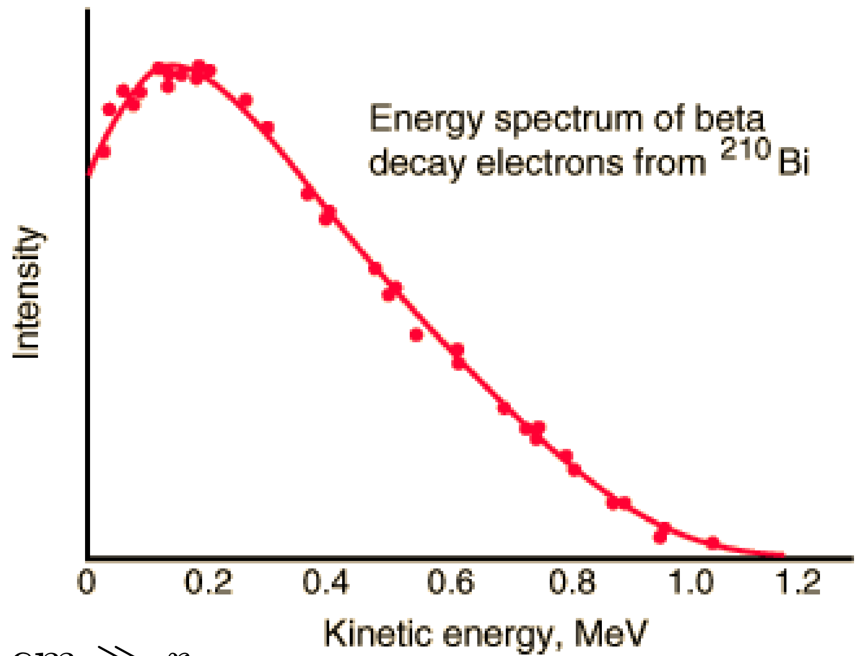
but mass of proton $m_p < m_n$ (mass of neutron)?!...

- ...possible if neutrons are not free (nuclear binding energy)
- This reaction is the main source of solar neutrinos:

$$4p \to {}^4\text{He} + 2e^+ + 2\nu_e + 26.7\ \text{MeV}$$

Fermi theory of β-decay

- The emitted electrons are (quasi) relativistic (kinetic energies from keV to few MeV). Neutrinos are relativistic



- Typical nuclear size $r_{\text{nucl}} \sim 10^{-13}$ cm.

- Typical wavelengths:

$$\frac{\hbar}{p_e} \sim \frac{\hbar}{p_{\bar{\nu}_e}} \sim \frac{\hbar}{100\,\mathrm{keV}} \sim 2\times10^{-10}\,\mathrm{cm} \gg r_{\text{nucl}}$$

(where $\hbar = 6.6 \times 10^{-19}$ keV·sec)

- Fermi: **assume** that there are some new interactions V_W. These interactions are small perturbations around free particle states $p, n, e, \bar{\nu}_e$

- As the wave lengths of two particles are much larger than the nuclear size, one can think that this interaction is "point-like" (4 particles meet at one point)

$$V_{if} = \langle i | V_W | f \rangle \approx \text{const} = G_F$$

- G_F is a new constant with a dimension EL^3 — **Fermi constant**. Its value should be determined experimentally

- Fermi: let us describe this process via 4-fermions $(n, p, e^-, \bar{\nu}_e)$ interaction.

- Take wave functions of 4 fermions: $\psi_p(x), \psi_n(x), \psi_e(x), \psi_{\bar{\nu}}(x)$. Let us call them ψ_A $(A = \{p, n, e^-, \bar{\nu}_e\})$ (notice that each of these wave-functions are spinors!!)

- In the absence of weak interactions each particles evolves freely with its own momentum and its own mass, m_A

$$i\partial_t \psi_A = \Big(\boldsymbol{\alpha} \cdot \boldsymbol{p} + \beta m_A\Big)\psi_A$$

- Build a **4-particles wave function** (depends on 4 variables)

$$\Psi(\boldsymbol{x}_1, \boldsymbol{x}_2, \boldsymbol{x}_3, \boldsymbol{x}_4|t) = \psi_n(\boldsymbol{x}_1, t)\psi_p(\boldsymbol{x}_2, t)\psi_e(\boldsymbol{x}_3, t)\psi_\nu(\boldsymbol{x}_4, t) \qquad (27)$$

- It obeys free Dirac equation

$$i\frac{\partial\Psi(\boldsymbol{x}_1,\boldsymbol{x}_2,\boldsymbol{x}_3,\boldsymbol{x}_4|t)}{\partial t}=\left[\mathcal{H}_{x_1}+\mathcal{H}_{\boldsymbol{x}_2}+\mathcal{H}_{\boldsymbol{x}_3}+\mathcal{H}_{\boldsymbol{x}_4}\right]\Psi(\boldsymbol{x}_1,\boldsymbol{x}_2,\boldsymbol{x}_3,\boldsymbol{x}_4|t) \tag{28}$$

where $\mathcal{H}_{x_i}=-i\boldsymbol{\alpha}\cdot\nabla_{x_i}+\beta m_A$

- The interaction is short-ranged i.e. all the particles should be **meeting at one point**:

$$V_W(\boldsymbol{x}_1,\boldsymbol{x}_2,\boldsymbol{x}_3,\boldsymbol{x}_4)=G_F\delta^{(3)}(\boldsymbol{x}_1-\boldsymbol{x}_2)\delta^{(3)}(\boldsymbol{x}_2-\boldsymbol{x}_3)\delta^{(3)}(\boldsymbol{x}_3-\boldsymbol{x}_4) \tag{29}$$

- Let us compute $\langle i|\,V_W\,|f\rangle$ where

$$\begin{aligned}|i\rangle&=|n\rangle &&\text{i.e.}\quad \Psi(\boldsymbol{x}_1,\boldsymbol{x}_2,\boldsymbol{x}_3,\boldsymbol{x}_4)=e^{i\boldsymbol{p}_n\boldsymbol{x}_1}\\ |f\rangle&=\left|p,e^-,\bar\nu_e\right\rangle &&\text{i.e.}\quad \Psi_f(\boldsymbol{x}_1,\boldsymbol{x}_2,\boldsymbol{x}_3,\boldsymbol{x}_4)=e^{i\boldsymbol{p}_p\boldsymbol{x}_2}e^{i\boldsymbol{p}_e\boldsymbol{x}_3}e^{i\boldsymbol{p}_\nu\boldsymbol{x}_4}\end{aligned} \tag{30}$$

we have

$$\langle i|\, V_W \,|f\rangle = \int d^3x_1 \dots d^3x_4 \,\Psi_i^* V_W(x_1,x_2,x_3,x_4)\Psi_f \qquad (31)$$

three δ-functions are removed by integrating over x_1, x_2, x_3, and we are left with

$$\langle i|\, V_W \,|f\rangle = G_F \int d^3\boldsymbol{x}_4 \, e^{(\boldsymbol{p}_p+\boldsymbol{p}_e+\boldsymbol{p}_\nu-\boldsymbol{p}_n)\cdot\boldsymbol{x}_4} = (2\pi)^3 G_F \delta^{(3)}(\boldsymbol{p}_p+\boldsymbol{p}_e+\boldsymbol{p}_\nu-\boldsymbol{p}_n) \qquad (32)$$

- ...putting the matrix element (32) into the formula

$$dW_{if} = 2\pi |V_{if}|^2 \delta(E_i - E - f) d\nu$$

- ...and integrating over the proton momentum $d^3\boldsymbol{p}_p$ and thus removing momentum delta-function, we will arrive to the expression

$$dW_{if}(p_e, p_{\bar{\nu}_e}) = 2\pi |V_{if}|^2 \delta(Q - E_e - E_{\bar{\nu}_e}) \frac{d^3 p_e}{(2\pi)^3} \frac{d^3 p_{\bar{\nu}_e}}{(2\pi)^3} \tag{33}$$

- when we compute the $|\langle i| V_W |f\rangle|^2$ we encountered expression like

$$\int d^3\boldsymbol{p}_e \, \delta^{(3)}(\boldsymbol{p}_p + \boldsymbol{p}_e + \boldsymbol{p}_\nu - \boldsymbol{p}_n)\delta^{(3)}(\boldsymbol{p}_p + \boldsymbol{p}_e + \boldsymbol{p}_\nu - \boldsymbol{p}_n) = \delta^{(3)}(0)$$

Using the well-known representation of the δ-function:

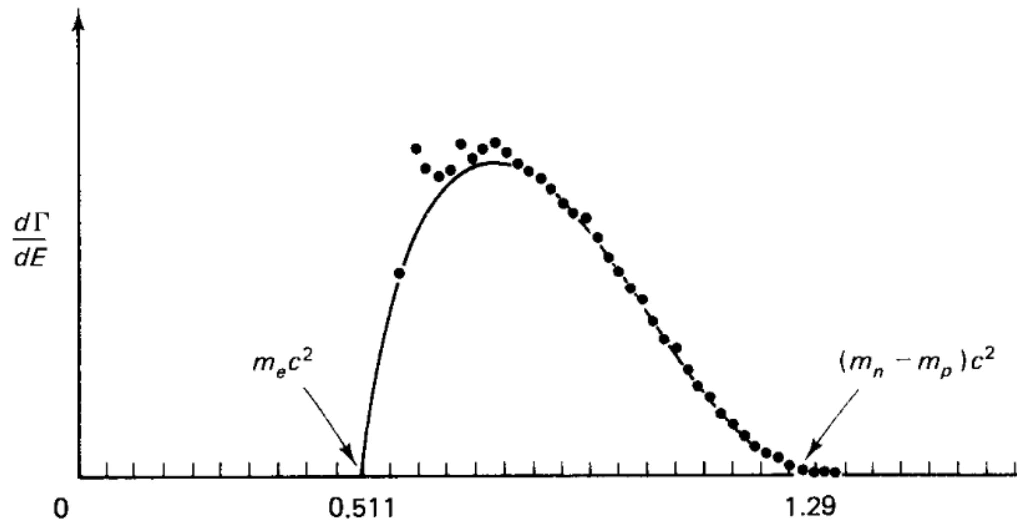
$$\delta^{(3)}(\boldsymbol{p}) = \int \frac{d^3\boldsymbol{x}}{(2\pi)^3} \, e^{i\boldsymbol{p}\cdot\boldsymbol{x}} \Rightarrow \delta^{(3)}(0) = \int \frac{d^3\boldsymbol{x}}{(2\pi)^3} = V$$

- Integrate over the momentum of $\bar{\nu}_e$ (it is not detected) and build a spectrum of electrons, using $d^3p_e = 4\pi p_e^2 dp_e = 4\pi\sqrt{E_e^2 - m_e^2}E_e dE_e$

$$dW(E_e) = \int \frac{d^3p_{\bar\nu_e}}{(2\pi)^3} dW_{if}(p_e, p_{\bar\nu_e}) = \frac{1}{2\pi^3} G_F^2 \sqrt{E_e^2 - m_e^2} E_e (Q - E_e)^2 dE_e \tag{34}$$

. . . which gets us the solution for the spectrum of electron β-decay:

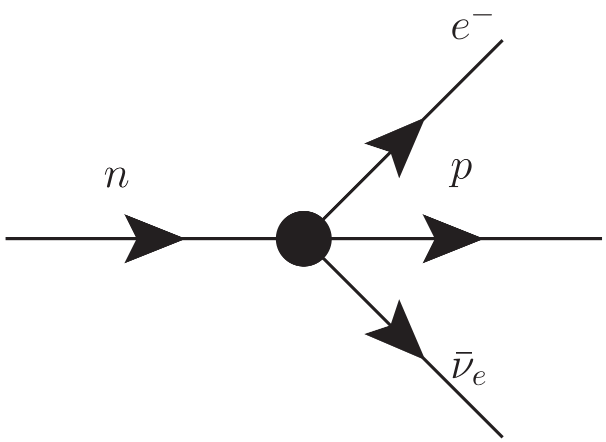
$$\frac{dW}{dE_e} = \frac{G_F^2}{2\pi^3}\sqrt{E_e^2 - m_e^2}E_e(Q - E_e)^2$$



- G_F – Fermi coupling constant. Value determined experimentally to be $G_F \approx 1.16 \times 10^{-5}$ GeV^{-2}

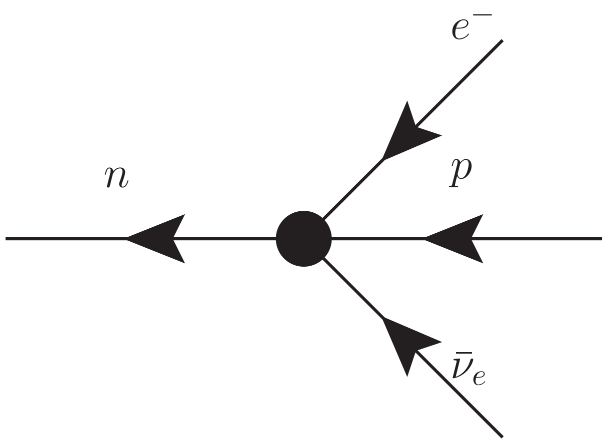
- Interaction (42) describes the process

$$n \to p + e^- + \bar{\nu}_e$$



- The same interaction also describes an **inverse process**:

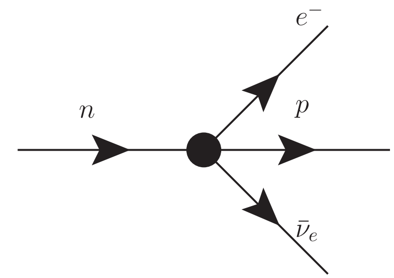
$$p + e^- + \bar{\nu}_e \to n$$



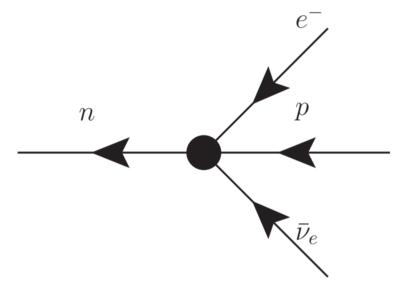
- The matrix elements of both processes are equal

$$|\langle n|\, V_W \,|p, e^-, \bar\nu_e\rangle|^2 = |\langle p, e^-, \bar\nu_e|\, V_W \,|n\rangle|^2$$

this is just a consequence of time-reversal symmetry



- however the probability for 3 particles to meet at 1 point of space-time with the correct value of momenta of all 3 particles is extremely small



- What other processes are mediated by the same 4-fermion interaction?

- If you take any particle from a **finite** state, C-conjugate it and move to the **initial** state. Such process is also possible

Proton–antineutrino scattering

- Let us compute the cross-section in the Fermi theory of the process

$$p + \bar{\nu}_e \to n + e^+$$

— the process that has been used to detect the neutrino for the first time!

- The computation proceeds analogously to Eqs. (30)–(32) with different initial states (e.g. $|i\rangle = |p, \bar{\nu}_e\rangle$, $|f\rangle = |n, e^+\rangle$). Proton is at rest $E_p = m_p$, $\boldsymbol{p}_p = 0$, anti-neutrino is relativistic $E_\nu = |\boldsymbol{p}_\nu|$

- Recall that the total cross-section is given by

$$\sigma_{tot}(E_\nu) = \int \frac{d^3\boldsymbol{p}_e}{(2\pi)^3} \frac{d^3\boldsymbol{p}_n}{(2\pi)^3} |V_{if}|^2 \delta(E_\nu + m_p - E_n - E_e) \qquad (35)$$

where the matrix element between the $|i\rangle$ and $|f\rangle$ states is equal to $V_{if} = G_F \delta^{(3)}(\boldsymbol{p}_\nu - \boldsymbol{p}_n - \boldsymbol{p}_e)$.

- as a result we obtain (we take $E_e \approx |\boldsymbol{p}_e|$ – relativistic positron)

$$
\begin{aligned}
\sigma_{tot}(E_\nu) =& 2\pi G_F^2 \int \frac{d^3\boldsymbol{p}_e}{(2\pi)^3}\delta(m_p + E_\nu - |\boldsymbol{p}_e| - \sqrt{m_n^2 + (\boldsymbol{p}_\nu - \boldsymbol{p}_e)^2}) \\
\approx& \frac{G_F^2}{\pi}\frac{E_\nu^2 m_p}{2E_\nu + m_p}
\end{aligned}
\tag{36}
$$

where we neglected $m_n - m_p$ difference and the positron mass, m_e

- From Eq. (35) we see that

$$
\sigma_{tot}(E_\nu) \approx \begin{cases} \dfrac{G_F^2}{\pi} E_\nu^2, & E_\nu \ll m_p \\ \dfrac{G_F^2}{2\pi}(E_\nu m_p), & E_\nu \gg m_p \end{cases}
\tag{37}
$$

notice that central of mass energy of the system “proton-at-rest” + neutrino is given by $E_{cm} = \sqrt{2m_p E_\nu}$ and therefore in the high-energy regime the cross-section (37) behaves as

$$\sigma_{tot} \approx G_F^2 E_{cm}^2 \tag{38}$$

- Using Eq. (37) for $E_\nu \sim 1$ MeV (i.e. $E_\nu \ll m_p \approx 1$ GeV) we find that

$$\sigma_{tot}(1\,\mathrm{MeV}) \approx 3 \times 10^{-17}\,\mathrm{GeV}^{-2} \approx 10^{-44}\,\mathrm{cm}^2 \tag{39}$$

using $1\,\mathrm{barn} \equiv 10^{-24}\,\mathrm{cm}^2 = 2.57 \times 10^3\,\mathrm{GeV}^{-2}$

- for comparison: photon scattering on non-relativistic electron (**Thomson cross-section**) is $\sigma_{Thomson} \sim 10^{-24}\,\mathrm{cm}^2$
- In 1934 Bethe & Peierls did the estimate of the cross-section (39)
- In 1942 Fermi had build the first nuclear reactor – source of large number of neutrinos ($\sim 10^{13}\mathrm{neutrinos/sec/cm}^2$)

Proton-antineutrino scattering

- Flux of neutrinos from e.g. a nuclear reactor can initiate β^+ decays in protonts of **water**

- Event rate:

$$R = \text{Flux} \times \sigma_{tot} \times N_{\text{protons}} = 10^{13} \frac{1}{\sec \cdot \text{cm}^2} 10^{-44}\,\text{cm}^2 \times 10^{23} \approx 10^{-8} \sec^{-1} \qquad (40)$$

- Positrons annihilate and two γ-rays and neutron were detected!

- The existence of anti-neutrino has been experimentally confirmed by Cowan & Reines in 1956

- Examples of the processes described by the 4-fermion interaction with the same V_W (or G_F):

$$\begin{aligned} p + e^- &\leftrightarrow n + \nu_e \\ n + e^+ &\leftrightarrow p + \bar{\nu}_e \end{aligned} \tag{41}$$

- So far we have totally ignored the fact that all 4 fermions are spinors.
- The easiest way to take this into account is to add an interaction Lagrangian to be added to the sum of 4 free Lagrangians of the form (4)
$$\mathcal{L}_{\text{Fermi}} = -\frac{G_F}{\sqrt{2}}[\bar{p}(x)\Gamma_\mu n(x)][\bar{e}(x)\Gamma^\mu \nu(x)] \tag{42}$$
here Γ_μ are some of the γ-matrices. They could be γ^μ, $\gamma^\mu\gamma_5$ or more complicated linear combinations of those, such as $\Gamma^\mu = \gamma^\mu(c_V + \gamma_5 c_A)$, etc.
- Because all 4 fermions are spinors, we will use equation similar to (4) and the expressions for differential cross-section will be

different. However, it is clear that the integral cross-section will have the form (38) because at high energies, when we can neglect the masses of all particles, σ_{tot} can only depend on E_{cm}

- What matrices to put there was decided by the experiments
- Other weak processes: $\mu^- \to e^- + \bar{\nu}_e + \nu_\mu$ or $\pi^+ \to \mu^+ + \nu_\mu$, led to generalization of Fermi Lagrangian:

$$\mathcal{L}_{\text{Fermi}} = -\frac{G_F}{\sqrt{2}}\Big[J^\dagger_{\text{lepton}}(x) + J^\dagger_{\text{hadron}}(x)\Big] \cdot \Big[J_{\text{lepton}}(x) + J_{\text{hadron}}(x)\Big]$$

where current J^μ has leptonic ($e^\pm$, $\mu^\pm$, ν, $\bar{\nu}$) and hadronic (p, n, π) parts