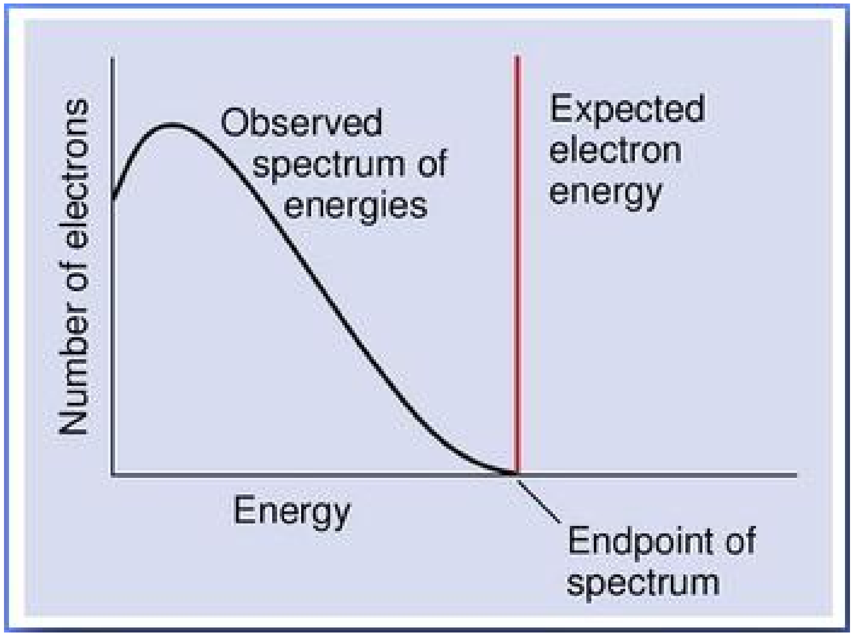
NEUTRINOS

Discovery of neutrino



- Observed ${}^{14}_{6}C \to {}^{14}_{7}N + e^{-}$
- Two body decay: the electron has the same energy (almost) $\Rightarrow$**not observed!**
- Energy is not conserved?

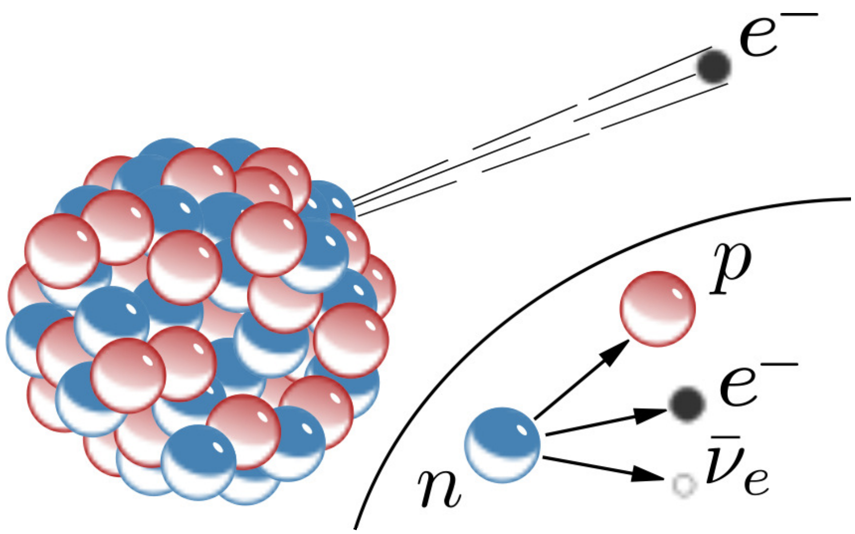
Pauli's letter, Dec. 4, 1930

...because of the "wrong" statistics of the N and Li^6 *nuclei and the continuous beta spectrum, I have hit upon a desperate remedy to save the "exchange theorem" of statistics and the law of conservation of energy. Namely, the* ***possibility that there could exist in the nuclei electrically neutral particles*** $\langle \dots \rangle$ ***which have spin 1/2 and obey the exclusion principle*** $\langle \dots \rangle$*. The continuous beta spectrum would then become understandable by the assumption that in beta decay a neutron is emitted in addition to the electron such that the sum of the energies of the neutron and the electron is constant...*

Neutrino

"I have done a terrible thing.
I invented a particle that cannot be detected."
W. Pauli

- Pauli (1930) called this new particle ~~**neutron**~~
- Chadwick discovered a massive nuclear particle in 1932 ⇒**neutron**
- Fermi renamed it into **neutrino** (italian “little neutral one”)



- β-decay is the decay of neutron

$$n \to p + e^- + \bar{\nu}_e$$

inside the nucleus

- Electron capture:

$$^{22}_{11}\text{Na} + e^- \to {}^{22}_{10}\text{Ne} + \nu_e$$

Here ν_e has **opposite spin** than that of $\bar{\nu}_e$!

- The process

$$p + e^- \to n + \nu_e$$

inside a nucleus

β^+-decay

- Also observed was β^+-decay

$$^{22}_{11}\text{Na} \to {}^{22}_{10}\text{Ne} + e^+ + \nu_e$$

Again the particle ν_e has the opposite spin to $\bar{\nu}_e$!

- Formally β^+ decay would come from the

$$p \stackrel{?}{\to} n + e^+ + \nu_e$$

but mass of proton $m_p < m_n$ (mass of neutron)?!...

- ...possible if neutrons are not free (nuclear binding energy)
- This reaction is the main source of solar neutrinos:

$$4p \to {}^4\text{He} + 2e^+ + 2\nu_e + 26.7\,\text{MeV}$$

Can neutrino be detected?

- Along with $p \stackrel{?}{\to} n + e^+ + \nu_e$ there can be a reaction $\bar{\nu}_e + p \to n + e^+$
  $\Rightarrow$energetic neutrino can cause β^+ decay of a stable nucleus

- In 1934 estimated that the cross-section for such a reaction is tremendously small: $\sigma \sim 10^{-43}\,\mathrm{cm}^2$ Bethe & Peierls

  for comparison: cross-section of photon scattering on non-relativistic electron (**Thomson cross-section**) is $\sigma_{Thomson} \sim 10^{-24}\,\mathrm{cm}^2$

- In 1942 Fermi had build the first nuclear reactor – source of large number of neutrinos ($\sim 10^{13}\mathrm{neutrinos/sec/cm^2}$)

- Flux of neutrinos from e.g. a nuclear reactor can initiate β^+ decays in protonts of **water**

- Positrons annihilate and two γ-rays and neutron were detected!

Discovered by Cowan & Reines in 1956

How to describe ν_e and $\bar{\nu}_e$

- Dirac equation describes evolution of 4-component spinor ψ_ν

$$(i\gamma^\mu \partial_\mu - m_\nu)\psi_\nu = 0$$

(where neutrinos mass $m_\nu \lll m_e$ but not necessarily zero)!

- This would predict that there should be particle and anti-particle (neutrino and **anti**-neutrino): $\psi_\nu = (\chi, \bar{\chi})$. Each of χ and $\bar{\chi}$ has spin $\uparrow$ and spin $\downarrow$ components

- Are χ and $\bar{\chi}$ distinct? So far people detected only neutral particles with spin up or down. Only two degrees of freedom?

- One possibility: one can put a solution of the Dirac equation, putting $\chi(\downarrow) = \bar{\chi}(\uparrow) \equiv 0$. This seems to be impossible, because even if one starts with such a solution, it will change over the course of evolution.

How to describe ν_e and $\bar{\nu}_e$

- This can be done **only** if $m_\nu = 0$. Indeed, if $m_\nu = 0$, then Dirac equation splits into two equations:

$$i\sigma^\mu \partial_\mu \begin{pmatrix} \chi(\uparrow) \\ \bar{\chi}(\downarrow) \end{pmatrix} = 0$$

  (and the one with interchanged spins).

- If this is true, then the prediction is that there are particles ν_e that always have definite spin projection and anti-neutrino with the opposite spin projection.

- These particles are **distinct** (there is an interaction that differentiates between two)

- **Alternatively**, **identify** $\chi(\uparrow) = \bar{\chi}(\downarrow)$ and vice versa. Overall, there are only two degrees of freedom:

$$\text{Majorana fermion} \qquad \psi_\nu = \begin{pmatrix} \chi \\ i\sigma_2\chi^* \end{pmatrix}$$

where the matrix $i\sigma_2 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ interchanges spin-up and spin-down components of the spinor χ^*

- The corresponding modification of the Dirac equation was suggested by Majorana (1937)

$$i\sigma^\mu\partial_\mu\chi - m_\nu i\sigma_2\chi^* = 0 \qquad (1)$$

Such equation is possible only for **trully neutral particles**

- Indeed, if the particle is not neutral, i.e. if there is a charge under which $\chi \overset{\hat{Q}}{\to} \chi e^{i\alpha Q}$ then χ^* has the opposite charge and equation (1) does not make sense (relates two objects with different transformation properties)

- Notice that when $m_\nu = 0$ there is no difference between Majorana equatino for the massless particle and massless 2-component Dirac equation

- In the same year when Cowan & Reines did their experiment, people were not abe to detect the same reactor neutrinos via

$$\bar{\nu}_e + {}^{37}_{17}\text{Cl} \not\longrightarrow \quad {}^{37}_{18}\text{Ar} + e^- \tag{2}$$

- The conclusion was that ν_e (that would be captured in reaction (2)) and $\bar{\nu}_e$ that **was** captured by proton in Cowan & Reines were different particles!

Muon

- Muon, discovered in cosmis rays — heavier “brother” of electron
- Mass: $m_\mu \approx 105$ MeV

Another type of neutrino

- Later it was observed that **muon** decays always via 3-body decay:

$$\mu^- \to e^- + \bar{\nu}_e + \nu_\mu$$

- ...but never via two-body decay, e.g.

$$\mu^- \to e^- + \gamma \qquad \leftarrow \textbf{not observed!}$$

or

$$\mu^- \to e^- + e^+ + e^- \qquad \leftarrow \textbf{not observed!}$$

- Pions decay to muons or electrons via two-body decay, emitting some neutral massless particle with the spin 1/2:

$$\pi^+ \to \mu^+ + \nu_\mu \qquad \pi^- \to \mu^- + \bar{\nu}_\mu$$
$$\pi^+ \to e^+ + \nu_e \qquad \pi^- \to e^- + \bar{\nu}_e$$

Universality of weak interactions

- Original Fermi theory (1934)

$$\mathcal{L}_{\text{Fermi}} = \frac{G_F}{\sqrt{2}}[\bar{p}(x)\gamma_\mu n(x)][\bar{e}(x)\gamma^\mu \nu(x)] \tag{3}$$

- Universality of weak interactions

before 1957!

$$\begin{aligned}\mathcal{L}_{\text{Fermi}} &= \frac{G_F}{\sqrt{2}}\underbrace{[\bar{p}(x)\gamma_\mu n(x)][\bar{e}(x)\gamma^\mu \nu_e(x)]}_{\beta-\text{decay}} \\ &+ \frac{G_F}{\sqrt{2}}\underbrace{[\bar{\mu}(x)\gamma^\mu \nu_\mu(x)[\bar{e}(x)\gamma^\mu \nu_e(x)]}_{\text{muon decay}} \\ &+ \frac{G_F}{\sqrt{2}}\underbrace{[\bar{e}(x)\gamma^\mu \nu_e(x)[\bar{e}(x)\gamma^\mu \nu_e(x)]}_{\text{electron-neutrino scattering}} \\ &\ldots (\text{pion decay, etc.})\end{aligned} \tag{4}$$

- All processes are governed by the same **Fermi coupling constant:** $G_F \approx 1.16 \times 10^{-5}\,\text{GeV}^{-2}$

Parity transformation

- **Parity transformation** is a discrete space-time symmetry, such that all spatial coordinates flip $\vec{x} \to -\vec{x}$ and time does not change $t \to t$

- **Show** that:

P-even	P-odd
time	position $\vec{x}$
angular momentum	momentum $\vec{p}$
mass density	Force
electric charge	electric current
magnetic field	electric field

- For fermions parity is related to the notion of **chirality**

- Dirac equation without mass can be split into two non-interacting parts

$$(i\gamma^\mu \partial_\mu - \cancel{m})\psi = \begin{pmatrix} \cancelto{0}{-m} & i(\partial_t + \vec{\sigma}\cdot\vec{\nabla}) \\ i(\partial_t - \vec{\sigma}\cdot\vec{\nabla}) & \cancelto{0}{-m} \end{pmatrix} \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix} = 0$$

Parity transformation

- **Show** that if particle moves only in one direction $\boldsymbol{p} = (p_x, 0, 0)$, then these two components $\psi_{L,R}$ are **left-moving** and **right-moving** along x-direction.

- One can define the $\gamma_5 = i\gamma_0\gamma_1\gamma_2\gamma_3$. In the above basis (Peskin & Schroeder conventions)':

$$\gamma_5 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

  and

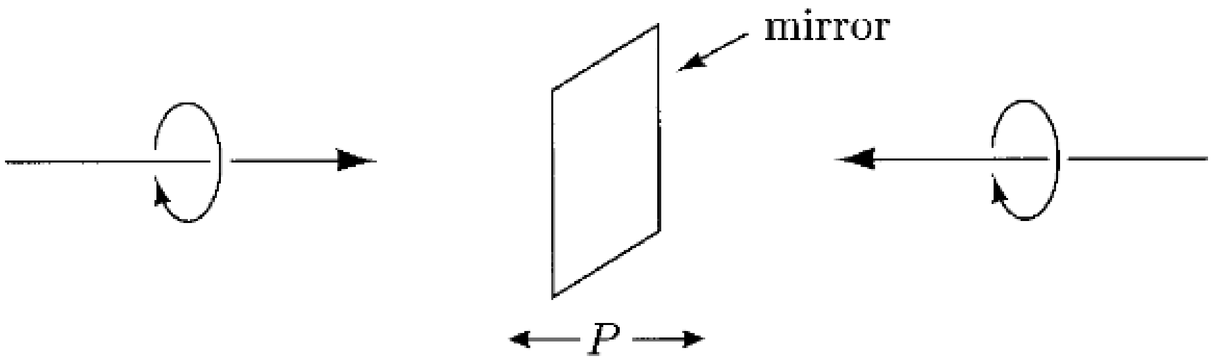
$$\gamma_5 \psi_{R,L} = \pm \psi_{R,L}$$

- **Show** that for massless fermions one can define a conserved quantity **helicity**: projection of spin onto momentum:

$$h \equiv \frac{(\boldsymbol{p} \cdot \boldsymbol{s})}{|\boldsymbol{p}|}$$

  Show that left/right chiral particles have definite helicity ± 1.

Parity transformation

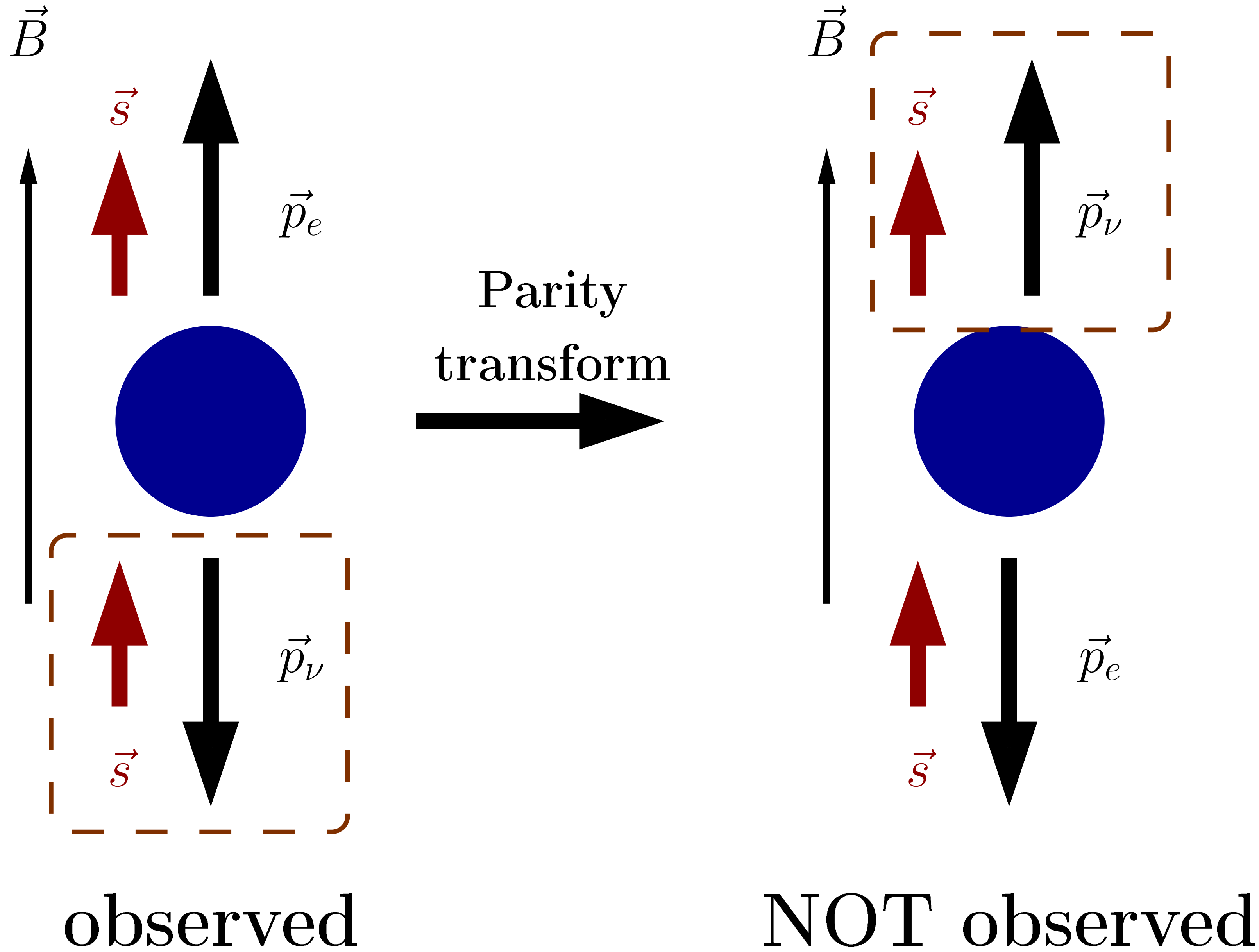


- For fermions: Left-Handed $\rightleftarrows$ Right-Handed. $|\psi_L(\vec{p})\rangle \xrightarrow{P} |\psi_R(-\vec{p})\rangle$.
- The neutrino being massless particle would have two states:
  - Left neutrino: spin **anti-parallel** to the momentum $\boldsymbol{p}$
  - Right neutrino: spin **parallel** to the momentum $\boldsymbol{p}$
- This has been tested in the experiment by Wu et al. in 1957

Parity violation in weak interactions

- Put nuclei into the strong magnetic field to align their spins
- Cool the system down (to reduce fluctuations, flipping spin of the Cobalt nucleus)
- The transition from ^{60}Co to ^{60}Ni has momentum difference $\Delta J = 1$ (spins of nuclei were known)
- Spins of electron and neutrino are parallel to each other
- Electron and neutrino fly in the opposite directions
- **Parity** flips momentum but does not flip angular momentum/spin/magnetic field

Parity violation in weak interactions



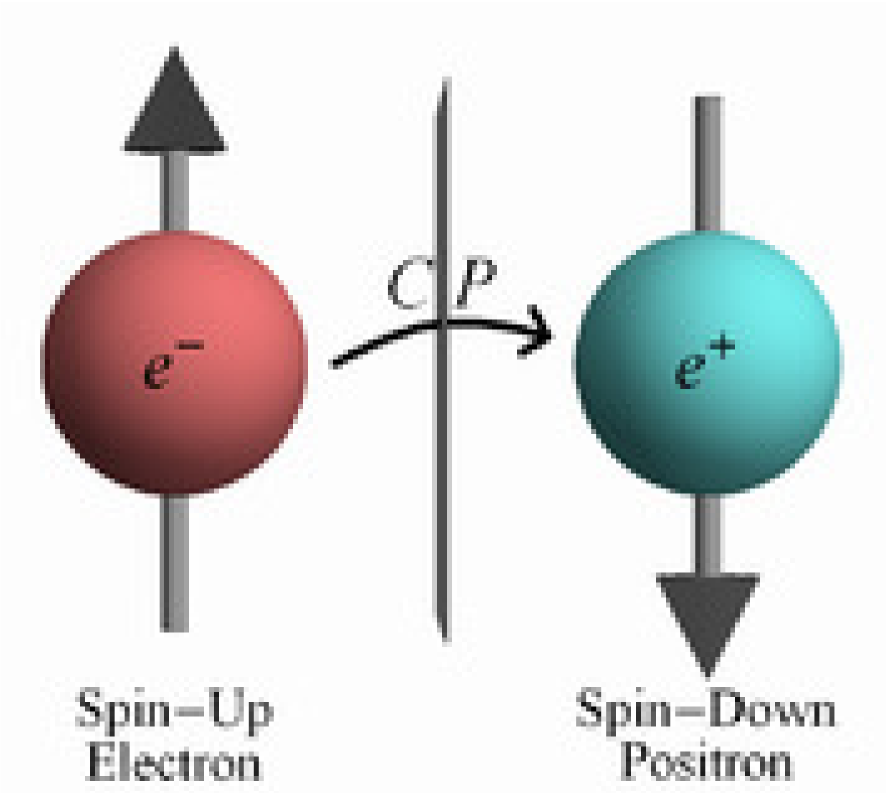
- This result means that neutrino always has spin anti-parallel to its momentum (**left-chiral particle**)

- Parity exchanges left and right chiralities. As neutrino is always left-polarized this means that

Particle looked in the mirror and did not see itself ???

- How can this be?

- Another symmetry, **charge conjugation** comes to rescue. Charge conjugation exchanges particle and anti-particle. Combine it with parity (**CP-symmetry**):



  - P: $|\nu_L\rangle \to |\nu_R\rangle$ – **impossible**
  - CP: $|\nu_L\rangle \to |\bar{\nu}_R\rangle$ – **possible**. Anti-neutrino exists and is always right-polarized
  - Life turned out to be more complicated. CP-symmetry is also broken

Parity and weak interactions

$$\mathcal{L}_{\text{Fermi}} = \frac{G_F}{\sqrt{2}} \underbrace{[\bar{p}(x)\gamma_\mu(1-\gamma_5)n(x)][\bar{e}(x)\gamma^\mu(1-\gamma_5)\nu_e(x)]}_{\beta-\text{decay}} \tag{5}$$

$$+ \frac{G_F}{\sqrt{2}} \underbrace{[\bar{\mu}(x)\gamma^\mu(1-\gamma_5)\nu_\mu(x)[\bar{e}(x)\gamma^\mu(1-\gamma_5)\nu_e(x)]}_{\text{muon decay}}$$

$$+ \frac{G_F}{\sqrt{2}} \underbrace{[\bar{e}(x)\gamma^\mu(1-\gamma_5)\nu_e(x)[\bar{e}(x)\gamma^\mu(1-\gamma_5)\nu_e(x)]}_{\text{electron-neutrino scattering}}$$

$$\ldots (\text{pion decay, etc.})$$

Only left (spin opposed to momentum) neutrinos and right (spin co-aligned with momentum) anti-neutrinos are produced or detected in weak interactions

The weak interactions conserve **flavour lepton numbers**

Detection of other neutrinos

- Muon neutrino ν_μ has been eventually detected via the process:

$$\mu \to e^- + \bar{\nu}_e + \nu_\mu$$
$$\hookrightarrow \nu_\mu + n \to p + \mu^-$$

- If the particle produced in muon decay were ν_e — it would not produce muon in the second reaction (but electron instead)

- Such a reaction was observed in 1962 by Lederman, Schwartz and Steinberger

- In (1975), the third lepton, τ, has been discovered. The third type of neutrino, ν_τ as found in (2000)

- To this date there has not been a single detection of $\bar{\nu}_\tau$, although we do believe in its existence

Fermion number conservation

- Recall $\mathcal{L} = \bar\psi \partial\!\!\!/ \psi + m\bar\psi\psi$ does not change if $\psi \to \psi e^{i\alpha}$
- Nöther theorem guarantees fermion number conservation:

$$J_F^\mu = \bar\psi\gamma^\mu\psi \qquad \partial_\mu J_F^\mu = 0$$

- If there are several **flavours** (ψ_i) then

$$\mathcal{L} = \sum_{i=1}^{N} \bar\psi_i \partial\!\!\!/ \psi_i + m_i\bar\psi_i\psi_i \tag{6}$$

- we have N conserved **fermion (flavour) numbers**

$$J_i^\mu = \bar\psi_i\gamma^\mu\psi_i \qquad \partial_\mu J_i^\mu = 0$$

As a consequence $J_F^\mu = \sum_i J_i^\mu$ is also conserved

Flavour lepton numbers

- Define flavor lepton numbers L_e, L_ν, L_τ:

	L_e	L_μ	L_τ			L_e	L_μ	L_τ
(ν_e, e^-)	+1	0	0		$(\bar\nu_e, e^+)$	-1	0	0
(ν_μ, μ^-)	0	+1	0		$(\bar\nu_\mu, \mu^+)$	0	-1	0
(ν_τ, τ^-)	0	0	+1		$(\bar\nu_\tau, \tau^+)$	0	0	-1

- Total lepton number is $L_{tot} = L_e + L_\mu + L_\tau$.

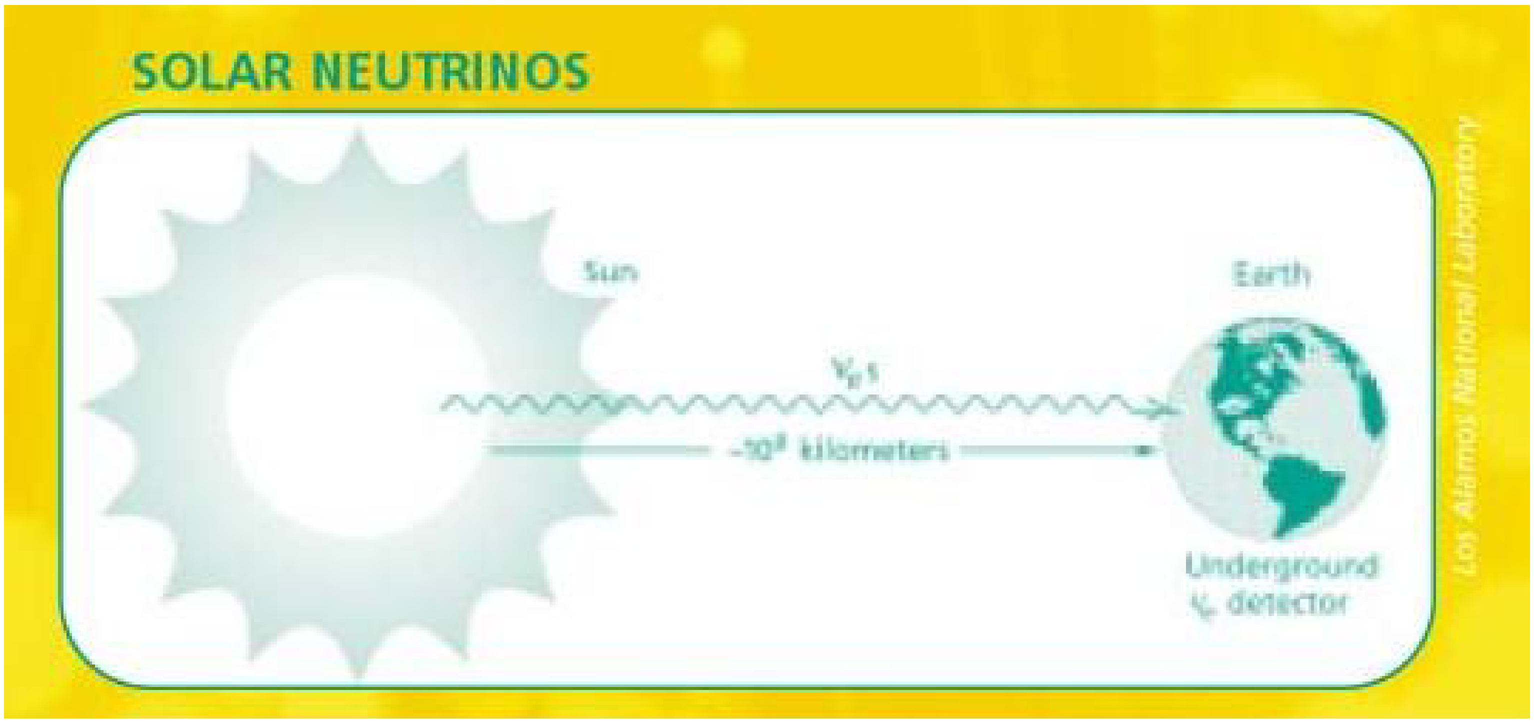
- **Symmetry of the Standard Model:** conserved **flavour lepton number** and **total lepton number**

- Fermi interactions respect this symmetry

$$\mathcal{L} = \begin{pmatrix} \bar\nu_e \\ \bar\nu_\mu \end{pmatrix} \underbrace{\left[i\partial\!\!\!/ \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} V^e_{\text{Fermi}} & 0 \\ 0 & V^\mu_{\text{Fermi}} \end{pmatrix} \right]}_{\text{weak interactions}} \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix}$$

Neutrino experiments

- ⋆ The **atmospheric** evidence: disappearance of ν_μ and $\bar{\nu}_\mu$ atmospheric neutrinos ($\nu_\mu \to \nu_\tau$) SuperKamioka
- ⋆ The **solar** evidence: deficit $\sim 50\%$ of solar ν_e ($\nu_e \to \nu_{\mu,\tau}$) SNO
- ⋆ The **reactor** evidence: disappearance of $\bar{\nu}_e$ produced by nuclear reactors. ***Back to neutrinos*** KamLAND



How is this possible if weak interactions conserve flavour?

Mass and charge eigenstates

- Define **charge eigenstates** as those where interaction term is diagonal

$$\mathcal{L} = \begin{pmatrix} \bar{\nu}_e \\ \bar{\nu}_\mu \end{pmatrix} \underbrace{\left[i\partial\!\!\!/ \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} V^e_{\text{Fermi}} & 0 \\ 0 & V^\mu_{\text{Fermi}} \end{pmatrix} \right]}_{\text{weak interactions}} \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix}$$

- For exampe, $V_{\text{Fermi}} \sim G_F n_e$ if neutrinos propagate in the medium with high density of electrons (interior of the Sun)

- If neutrinos have mass, the mass term is not necessarily diagonal in this basis:

$$\mathcal{L} = \begin{pmatrix} \bar{\nu}_e \\ \bar{\nu}_\mu \end{pmatrix} \underbrace{\left[i\partial\!\!\!/ \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} V^e_{\text{Fermi}} & 0 \\ 0 & V^\mu_{\text{Fermi}} \end{pmatrix} \right]}_{\text{weak interactions}} \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} + \begin{pmatrix} \bar{\nu}_e \\ \bar{\nu}_\mu \end{pmatrix} \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix} \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix}$$

Mass and charge eigenstates

- One can define **mass eigenstates** such that the kinetic plus mass term is diagonal in this basis

$$\mathcal{L} = \begin{pmatrix} \bar{\psi}_1 \\ \bar{\psi}_2 \end{pmatrix} \left[i\partial\!\!\!/ \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix} \right] \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} + \begin{pmatrix} \bar{\psi}_1 \\ \bar{\psi}_2 \end{pmatrix} \begin{pmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{pmatrix} \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$$

- A unitary transformation rotates between these two choices of basis

$$\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \underbrace{\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}}_{\text{matrix } U} \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$$

Neutrino oscillations

- Consider the simplest case: two flavours, two mass eigen-states. Matrix U is parametrized by one **mixing angle** θ

$$\begin{aligned} |\nu_e\rangle &= \cos\theta\,|1\rangle + \sin\theta|2\rangle \\ |\nu_\mu\rangle &= \cos\theta|2\rangle - \sin\theta|1\rangle \end{aligned}$$

- Let take the initial state to be ν_e (created via some weak process) at time $t = 0$:

$$|\psi_0\rangle = |\nu_e\rangle = \cos\theta\,|1\rangle + \sin\theta\,|2\rangle$$

- Then at time $t > 0$

$$|\psi_t\rangle = e^{-iE_1 t}\cos\theta\,|1\rangle + \sin\theta\,|2\rangle\,e^{-iE_2 t}$$

- We detect the particle later via another weak process (e.g. $\nu_? + n \to p + \mu^-/e^-$)

Neutrino oscillations

- The probability of conversion $\nu_e \to \nu_\mu$ is given by

$$P(\nu_e \to \nu_\mu) = |\langle \nu_\mu | \psi_t \rangle|^2 = \sin^2(2\theta) \sin^2 \left(\frac{(E_2 - E_1)t}{2} \right)$$

- The probability to detect ν_e is give by

$$P(\nu_e \to \nu_e) = |\langle \nu_e | \psi_t \rangle|^2 = \cos^2(2\theta) \sin^2 \left(\frac{(E_2 - E_1)t}{2} \right)$$

Fermion number conservation?

- Apparent violation of flavour lepton number for neutrinos **can be explained** by the presene of the non-zero neutrino mass

$$\mathcal{L} = \begin{pmatrix} \bar\nu_e \\ \bar\nu_\mu \\ \bar\nu_\tau \end{pmatrix} \underbrace{\left[i\partial\!\!\!/ - V_{\text{Fermi}} \right]}_{\text{conserves flavour number}} \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} + \begin{pmatrix} \bar\nu_e \\ \bar\nu_\mu \\ \bar\nu_\tau \end{pmatrix} \begin{pmatrix} m_{11} & m_{12} & \dots \\ m_{21} & m_{22} & \dots \\ \dots & \dots & \dots \end{pmatrix} \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}$$

- In this case only **one** fermion current (total lepton fermion number) is conserved:

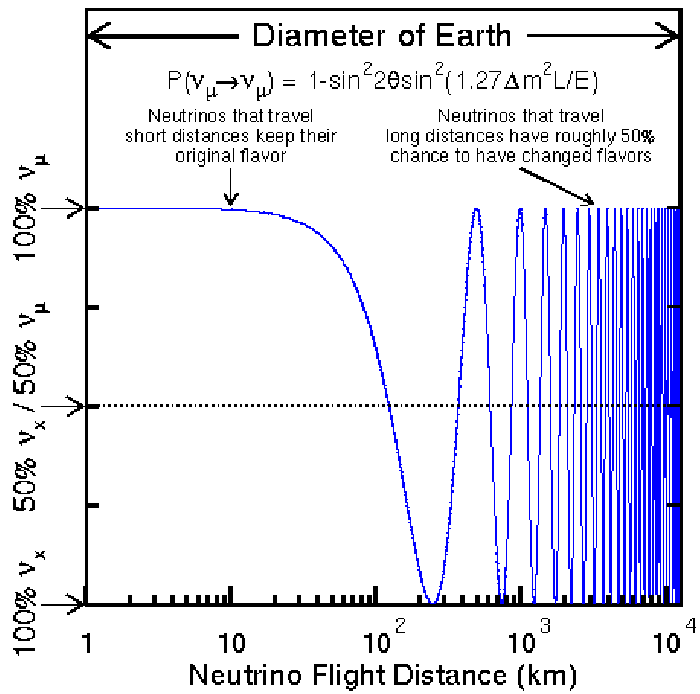
$$J^\mu = \sum_{i=e,\mu,\tau} \bar\nu_i \gamma^\mu \nu_i \tag{7}$$

while any independent $J_i^\mu = \bar\nu_i\gamma^\mu\nu_i$ is not conserved.

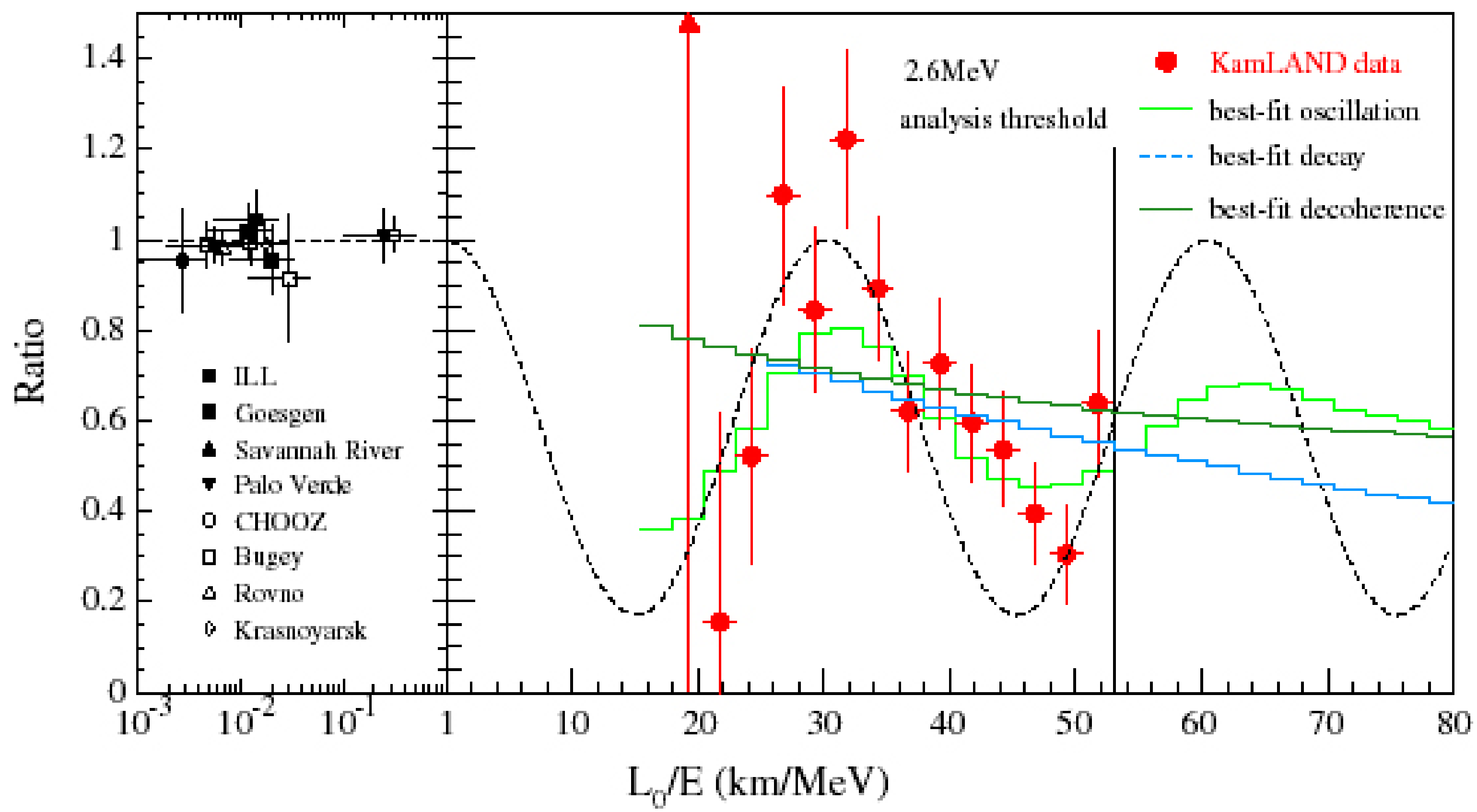
- The prediction is: neutrinos **oscillate**, i.e. probability to observe a given flavour changes with the distances travelled:

$$P_{\alpha\to\beta} = \sin^2(2\theta)\,\sin^2\left(1.267\frac{\Delta m^2 L}{E}\frac{\mathrm{GeV}}{\mathrm{eV}^2\,\mathrm{km}}\right) \quad (8)$$

Mass difference: Δm^2, Neutrino energy: E (keV-MeV in stars, GeV in air showers, etc. Distance traveled: L



- Eq. (8) predicts that probability oscillates as a function of the ratio E/L. This is indeed observed:



in this plot the distance between reactor and detector and energy is different, therefore E/L is different

3 neutrino generations

- Neutrino experiments determine **two** mass splittings between **three** mass eigenstates (m_1, m_2, m_3): $\Delta m^2_{\rm solar} = 7.6 \times 10^{-5}\ {\rm eV}^2$ and $|\Delta m^2_{\rm atm}| = 2.4 \times 10^{-3}\ {\rm eV}^2$

- A 3×3 unitary transformation U relates mass eigenstates (ν_1, ν_2, ν_3) to flavour eigenstates

$$\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix} = \begin{pmatrix}U_{e1} & U_{e2} & U_{e3}\\ U_{\mu1} & U_{\mu2} & U_{\mu3}\\ U_{\tau1} & U_{\tau2} & U_{\tau3}\end{pmatrix}\begin{pmatrix}\nu_1\\ \nu_2\\ \nu_3\end{pmatrix}$$

- Any unitary 3×3 martix has 9 real parameters:

$$U = Exponent\left[i\begin{pmatrix}\lambda_1 & |u_{12}|e^{i\delta_{12}} & |u_{13}|e^{i\delta_{13}}\\ |u_{12}|e^{-i\delta_{12}} & \lambda_2 & |u_{23}|e^{i\delta_{23}}\\ |u_{13}|e^{-i\delta_{13}} & |u_{23}|e^{-i\delta_{23}} & \lambda_3\end{pmatrix}\right]$$

How many of them can be measured in experiments?

Neutrino mixing matrix

- **Recall** that neutrinos $\nu_{e,\mu,\tau}$ couple to charged leptons $\Rightarrow$ Invariant under $\nu_e \to \nu_e e^{i\alpha}$ simultaneously with $e^- \to e^- e^{i\alpha}$, etc.
- All other terms in the Lagrangian have the form $\bar\psi \not{D} \psi$ or $m\bar\psi\psi$ — i.e. are invariant if $\psi \to \psi e^{i\alpha}$ (here ψ is any of $\nu_e, \nu_\mu, \nu_\tau, e, \mu, \tau$)
- Additionally, we can rotate each of the $\nu_{1,2,3}$ by an independent phase
- 5 of 9 parameters of the mixing matrix U can be absorbed in the redefinitions of $\nu_{1,2,3}$ and $\nu_{e,\mu,\tau}$ (6th phase does is overall redefinition of all fields – does not change U).

Neutrino mixing matrix

- The rest $9-5=4$ parameters are usually chosen as follows: **3 mixing angles** $\theta_{12},\theta_{23},\theta_{13}$ and **1 phase** ϕ (since 3×3 real orthogonal matrix has 3 parameters only)

$$U=\begin{pmatrix} c_{12}c_{13} & c_{13}s_{12} & s_{13} \\ -c_{23}s_{12}e^{i\phi}-c_{12}s_{13}s_{23} & c_{12}c_{23}e^{i\phi}-s_{12}s_{13}s_{23} & c_{13}s_{23} \\ s_{23}s_{12}e^{i\phi}-c_{12}c_{23}s_{13} & -c_{12}s_{23}e^{i\phi}-c_{23}s_{12}s_{13} & c_{13}c_{23}\end{pmatrix} \tag{9}$$

where one denotes $\cos\theta_{12}=c_{12}$, $\sin\theta_{23}=s_{23}$, etc.

Three rotations plus **one** phase ϕ:

$$U=\begin{pmatrix}1 & 0 & 0\\ 0 & \cos\theta_{23} & \sin\theta_{23}\\ 0 & -\sin\theta_{23} & \cos\theta_{23}\end{pmatrix}\begin{pmatrix}\cos\theta_{13} & 0 & e^{-i\phi}\sin\theta_{13}\\ 0 & 1 & 0\\ -e^{i\phi}\sin\theta_{13} & 0 & \cos\theta_{13}\end{pmatrix}\begin{pmatrix}\cos\theta_{12} & \sin\theta_{12} & 0\\ -\sin\theta_{12} & \cos\theta_{12} & 0\\ 0 & 0 & 1\end{pmatrix}$$

Problems for mixing matrix U

1. Show that for two flavour and two mass eigenstates the matrix U has **1** real free parameter (a **mixing angle**)

2. Show that if any of the angle θ_{12}, θ_{23} or θ_{13} is equal to zero, the matrix U can be chosen real

- **Charge conjugation** C : Particle $\rightleftarrows$ Antiparticle

  In general, all elementary particles can be divided into three groups:

  – **Truly neutral**, like photon, intermediate Z-boson, Majorana neutrino. They do not carry any charges
  – Particles: like proton, neutron, and electron
  – Antiparticles: antiproton, antineutron, and positron

- Naturally, a matter is a substance which consists of particles, and antimatter is a substance consisting of antiparticles.

- **Parity** P : Left-Handed $\rightleftarrows$ Right-Handed. $|\nu(\vec{p})\rangle \xrightarrow{P} |\nu(-\vec{p})\rangle$.

  Parity is broken in weak interactions (**only** left neutrinos and right antineutrinos participate in weak interactions)

- **Time reversal** T : $P_{\alpha\to\beta} \xrightarrow{T} P_{\beta\to\alpha}$
- **CP** : $|\nu(\vec{p})\rangle \xrightarrow{CP} |\bar{\nu}(-\vec{p})\rangle$

$$P_{\alpha\to\beta} \xrightarrow{CP} P_{\bar{\alpha}\to\bar{\beta}}$$

CP was believed to be the exact symmetry of nature after parity violations were discovered

- However: CP-violation in kaon decays (**1964** Cronin, Fitch,...) In a small fraction of cases ($\sim 10^{-3}$), long-lived K_L (a mixture of K^0 and $\bar{K}^0$ decays into pair of two pions, what is forbidden by CP-conservation.
- **If** CP were exact symmetry, an equal number of K^0 and $\bar{K}^0$ would produce an equal number of electrons and positrons in the reaction

$$K^0 \to \pi^- e^+ \nu_e, \quad \bar{K}^0 \to \pi^+ e^- \bar{\nu}_e,$$

- However, the number of positrons is somewhat larger ($\sim 10^{-3}$) than the number of electrons.

- **CPT**: $P_{\alpha\to\beta} \xrightarrow{CPT} P_{\bar\beta\to\bar\alpha}$

  CPT theorem: particles and antiparticles have the same mass, the same lifetime, but *all* their charges (electric, baryonic, leptonic, etc) are opposite.

- All known processes conserve CPT

Problems for discrete symmetries

1. s-quark carries a quantum number called "strangness" ($S\,|s\rangle = -1\,|s\rangle$). $K^+ = |u\bar{s}\rangle$. What is charge conjugated of K^+?

2. Neutral kaon $K^0 = |d\bar{s}\rangle$.

3. Is K^0 a truly neutral particle (i.e. does it coincide with its own anti-particle) ?

4. Is π^0 meson a truly neutral particle? Why?

5. Violation of P and CP are experimentally observed, while all processes are CPT symmetric. What other discrete symmetries are broken as a consequence of these experimental facts?

CP-violation in neutrino oscillations

- The matrix that rotates mass to flavour eigenstates:

$$U = \begin{pmatrix} c_{12}c_{13} & c_{13}s_{12} & s_{13} \\ -c_{23}s_{12}e^{i\phi} - c_{12}s_{13}s_{23} & c_{12}c_{23}e^{i\phi} - s_{12}s_{13}s_{23} & c_{13}s_{23} \\ s_{23}s_{12}e^{i\phi} - c_{12}c_{23}s_{13} & -c_{12}s_{23}e^{i\phi} - c_{23}s_{12}s_{13} & c_{13}c_{23} \end{pmatrix}$$

- $\nu_\alpha = U_{\alpha i}\nu_i$ (α is a **flavour index** e, μ, τ and i is the **mass index** $1, 2, 3$).

- Then probability $P_{\nu_\alpha \to \nu_\beta} \propto \left|\sum_i U_{ai} U^*_{i\beta}\right|^2$

- **CP-violation** in neutrino oscillations?

$$P_{\nu_\alpha \to \nu_\beta} \neq P_{\bar\nu_\alpha \to \bar\nu_\beta} \Longleftrightarrow \underbrace{\mathrm{Im}\left(\sum_{k<j} U_{\alpha k} U^*_{k\beta} U^*_{\alpha j} U_{j\beta}\right)}_{\propto \sin\phi} \neq 0$$

Neutrino mass from the point of view of the Standard Model?

Majorana mass terms

- Mass eigenstates $\nu_{1,2,3}$ are **freely propagating massive fermions**
- **Majorana mass term:**

$$\mathcal{L}_{\text{Majorana}} = \begin{pmatrix} \nu_1^c \\ \nu_2^c \\ \nu_3^c \end{pmatrix} \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} \tag{10}$$

m_1, m_2, m_3 can be complex

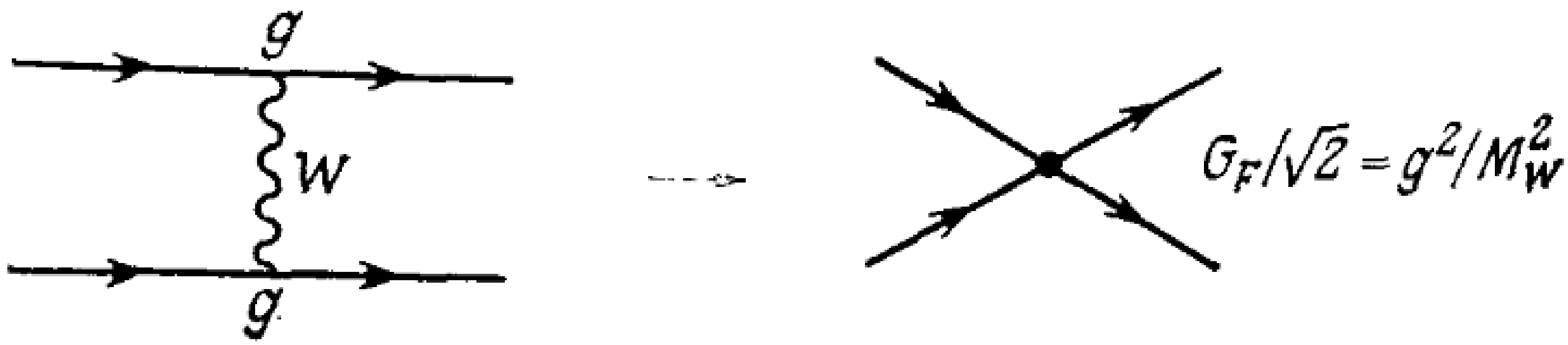
- ...or in terms of charge eigenstates:

$$\mathcal{L}_{\text{Majorana}} = M_{\alpha\beta} \bar{\nu}_\alpha \nu_\beta, \qquad \alpha, \beta = e, \mu, \tau$$

matrix $M_{\alpha\beta}$ is non-diagonal

- Can such a term exist in the Standard Model?

W-boson interactions



- Recall that Fermi-interactions are just a low energy limit of the interaction, mediated by a **vector boson** :

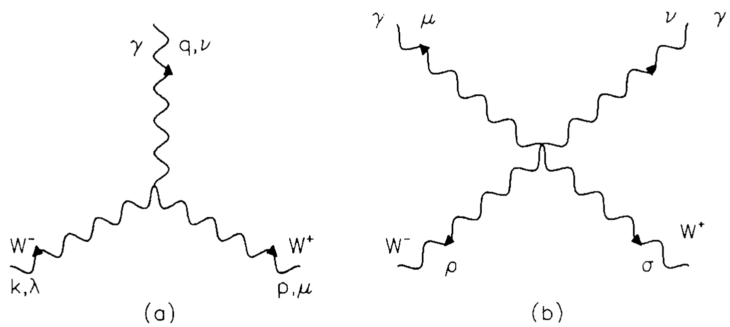
$$\mathcal{L}_{\text{Fermi}} \to \mathcal{L}_W = g(W_\mu^+ J^\mu + h.c.)$$

1957

- The $W^\pm$ are charged vector bosons and the intraction currents have lepton and hadron contributions:

$$J^\lambda_{\rm lepton} = \bar\nu_e\gamma^\lambda(1-\gamma_5)e + \bar\nu_\mu\gamma^\lambda(1-\gamma_5)\mu + \dots$$
$$J^\lambda_{\rm hadron} = \bar u\gamma^\lambda(1-\gamma_5)d + \text{other quarks}$$

- Charged nature of the W-bosons leads to two new interaction vertices involving photon.



notice that the diagram (a) depends both minimal term e that has the structure of $eW\partial WA$ and non-minimal term of the form $e\kappa WW\partial A$

- The interaction vertex $WW\gamma$ has the following form (for the choice of momentum as shown in Fig. (a) above)

$$V_{\lambda\mu\nu}(k,p,q) = \boldsymbol{e}\,\underbrace{(k-p)_\nu\eta_{\mu\lambda} + (p-q)_\lambda\eta_{\mu\nu} + (q-k)_\mu\eta_{\nu\lambda}}_{\equiv V^{\mathsf{YM}}_{\lambda\mu\nu}(k,p,q)} + \boldsymbol{e}(1-\boldsymbol{\kappa})(q_\lambda\eta_{\mu\nu} - q_\mu\eta_{\nu\lambda}) \tag{11}$$

- The interaction vertex $WW\gamma\gamma$ has the structure independent on κ, **but proportional on e^2 rather than e.**

$$V_{\mu\nu\rho\sigma} = -e^2(2\eta_{\mu\nu}\eta_{\rho\sigma} - \eta_{\mu\rho}\eta_{\nu\sigma} - \eta_{\mu\sigma}\eta_{\nu\rho}) \tag{12}$$

Z-boson

- In addition to W there is another massive **neutral** vector boson

- New vector boson couples to electrons **and to neutrinos** in the **parity violating** way and that also couples to W^+W^-.

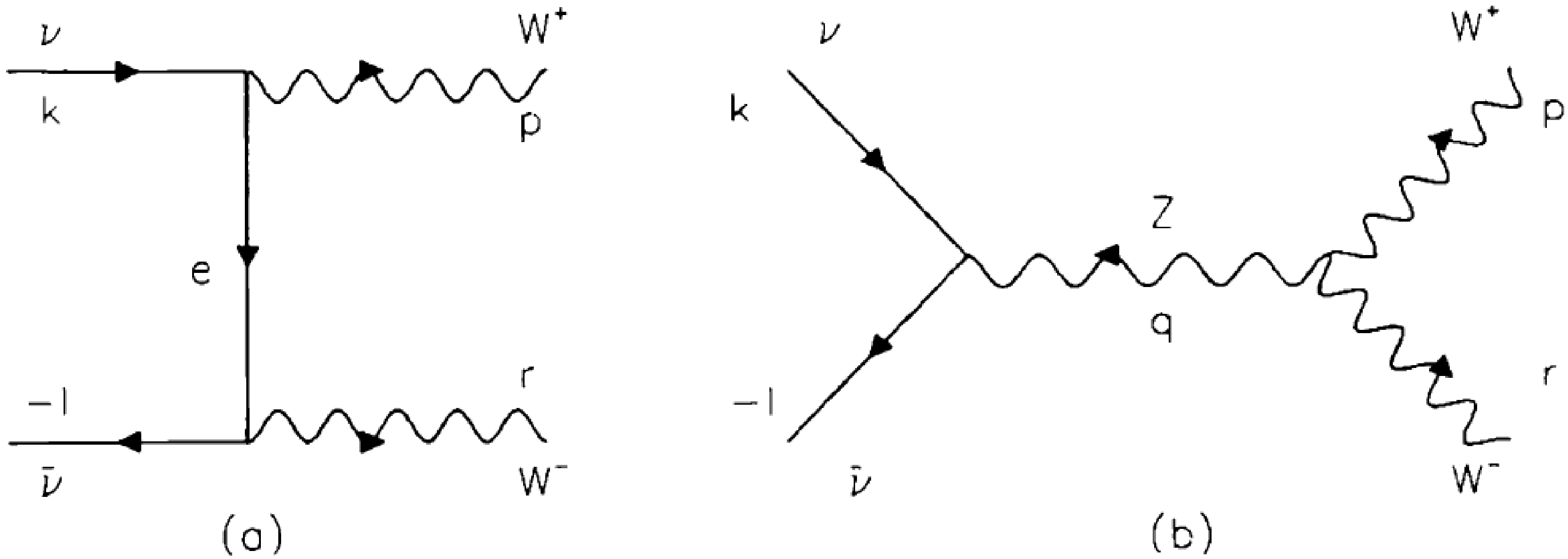
- New boson (**Z-bosons**) interacts with ν:

$$\mathcal{L}_{\bar{\nu}\nu Z} = \frac{1}{2}\boldsymbol{g_{\bar{\nu}\nu Z}}\bar{\nu}\gamma^{\mu}(1-\gamma_5)\nu Z_{\mu} \tag{13}$$

- New boson also interacts with W^+W^- and the vertex WWZ is similar to the vertex $WW\gamma$

- As result there are two processes contributing to $\bar{\nu}\nu \to W^+W^-$ scattering

Z-boson



Interactions of intermediate vector bosons

- The kinetic term for W-boson is

$$\mathcal{L}_W = -\frac{1}{2}(D_\mu W_\nu^- - D_\mu W_\nu^-)(D_\mu W_\nu^+ - D_\mu W_\nu^+) + \frac{M_W^2}{2} W_\mu^+ W_\mu^- + eF^{\mu\nu} W_\mu^+ W_\nu^- \quad (14)$$

where

$$D_\mu W_\nu^\pm = (\partial_\mu \mp ieA_\mu) W_\nu^\pm$$

- Similarly for Z-boson

$$\mathcal{L}_Z = -\frac{1}{2}(\partial_\mu Z_\nu - \partial_\nu Z_\mu)^2 + \partial ZWW + ZZWW \quad (15)$$

Symmetry between e^- and ν_e?

- Consider 2 different fermion fields $\psi^{(1)}$ and $\psi^{(2)}$ which are physically equivalent for some interaction (good historical example is n and p for strong interactions).

- The Dirac equation is

$$(i\gamma^\mu\partial_\mu - m)\,\psi^{(1)} + (i\gamma^\mu\partial_\mu - m)\,\psi^{(2)} + V_{int}(\psi^{(1)}) + V_{int}(\psi^{(2)}) \quad (16)$$

- We can compose two-component field $\vec{\Psi} = \begin{pmatrix}\psi^{(1)}\\ \psi^{(2)}\end{pmatrix}$ and rewrite the Dirac equation using $\vec{\Psi}$ as

$$(i\gamma^\mu\partial_\mu - m)\,\vec{\Psi} + \mathcal{L}_{int}(\vec{\Psi}) = 0 \quad (17)$$

- Probability

$$P = \int d^3x \left[\bar{\psi}^{(1)}(x)\gamma^0\psi^{(1)}(x) + \bar{\psi}^{(2)}\gamma^0\psi^{(2)}\right] = \int d^3x\, \vec{\Psi}^+\vec{\Psi}$$

Symmetry between e^- and ν_e?

and Dirac equation (17) are invariant under global transformations:

$$\vec{\Psi}(x) \to \vec{\Psi}'(x) = U\vec{\Psi}(x) \tag{18}$$

that leaves the "length" of the two-dimensional **isovector** $\vec{\Psi}$ invariant.

- Such transformation is called **unitary transformation** and the matrix U in Eq. is 2×2 complex matrix which obeys conditions $U^+U = \mathbb{1}$ and $\det(U) = 1$.

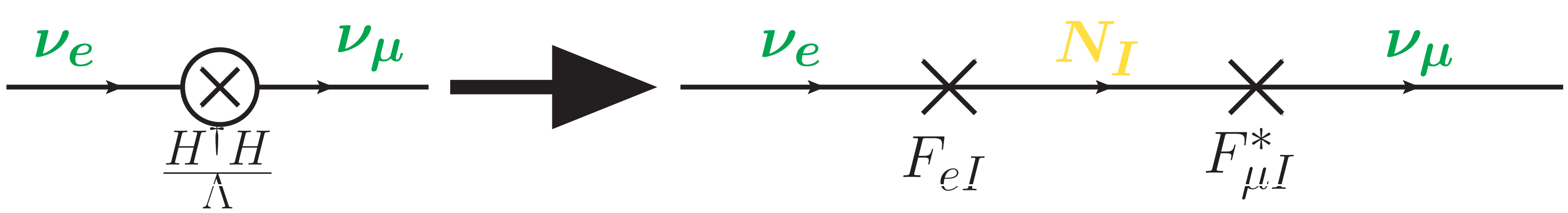
Majorana mass term

- Majorana mass term couples ν and its charge conjugated

- However, neutrino is a part of the SU(2) doublet $L = \begin{pmatrix} \nu \\ e \end{pmatrix}$ and therefore a Majorana mass term

$$\bar{\nu}^c \nu \to \frac{(\bar{L} \cdot H)(L \cdot H)}{\Lambda} \tag{19}$$

- This is "operator of dimension 5" (similar to G_F)



and it means that a new particle is present

Problems on neutrino mass term

1. Write the Dirac equation for fermions $\psi_i = (\nu_i, \nu_{iR})$, $i = 1, 2, 3$

2. The Majorana equation can be written for 2-component fermion as

$$i\sigma^\mu \partial_\mu \nu + m\nu^c = 0 \tag{20}$$

   (where σ are Pauli matrices, $\sigma_0 = \mathbf{1}$ and $\nu^c = i\sigma_2 \nu^*$)

   Show that its Lagrangian is Lorentz invariant (hint: use expression (10) as a mass term).

3. Show that the mass m in (20) can be complex number, yet the fermion ν has the dispersion relation $E^2 = \vec{p}^2 + |m|^2$

4. Show that rotation of each of the $\nu_{1,2,3}$ by a phase leaves the term (15) invariant (if one performs additional rotation of ν_{iR})

Problems on neutrino mass term

5. Show that rotation of each of the $\nu_{1,2,3}$ changes phase of the masses in (10) (which does not affect propagation in view of the exercise 3).

6. Show that of three Majorana masses in (10) **one** can be made real without changing the form of the matrix U in (16).

See-saw Lagrangian

Add right-handed neutrinos N_I to the Standard Model

$$\mathcal{L}_{\text{right}} = i\bar{N}_I \partial\!\!\!/ N_I + \begin{pmatrix} \bar{\nu}_e \\ \bar{\nu}_\mu \\ \bar{\nu}_\tau \end{pmatrix} \underbrace{\begin{pmatrix} F\langle H \rangle \end{pmatrix}}_{\text{Dirac mass } M_D} \begin{pmatrix} N_1 \\ N_2 \\ \dots \end{pmatrix} + \begin{pmatrix} N_1^c \\ N_2^c \\ \dots \end{pmatrix} \underbrace{\begin{pmatrix} M \end{pmatrix}}_{\text{Majorana mass}} \begin{pmatrix} N_1 \\ N_2 \\ \dots \end{pmatrix}$$

$\nu_\alpha = \tilde{H} L_\alpha$, where L_α are left-handed lepton doublets

- Active masses are given via usual **see-saw formula**:

$$(m_\nu) = -m_D \frac{1}{M_I} m_D^T \qquad ; \qquad m_D \ll M_I$$

- Neutrino mass matrix – **7 parameters**. Dirac+Majorana mass matrix – **11 (18) parameters** for 2 (3) sterile neutrinos. **Two** sterile neutrinos are enough to fit the neutrino oscillations data.

Problems about see-saw Lagrangian

1. Demonstrate that knowing the masses of all neutrinos does not allow to fix the scale of masses m_D and M_M.

2. Consider the Lagrangian with only one flavour and introduce one singlet right handed neutrino ν_R, and add both Majorana mass term to it and Dirac mass term via Higgs mechanism

$$\mathcal{L}_{\text{seesaw}} = \mathcal{L}_{SM} + \lambda_N \bar{L}_e H^c (\nu_R) + \frac{1}{2} M_M (\bar{\nu}_R^c)(\nu_R) + \text{h.c.} \, . \tag{21}$$

   Suppose, that the Dirac mass $m_D = \lambda_N v$ is much smaller than Majorana mass M_M, $m_D \ll M_M$. Find the spectrum (mass eigenstates) in (21). Identify linear combinations of ν and N that are mass **mass eigenstates** and rewrite the Lagrangian in this basis. Do not forget that the left double L_e participates in electric and weak interactions.

3. Generalize the above see-saw Lagrangian (21) for the number of SM lepton flavors other than one.

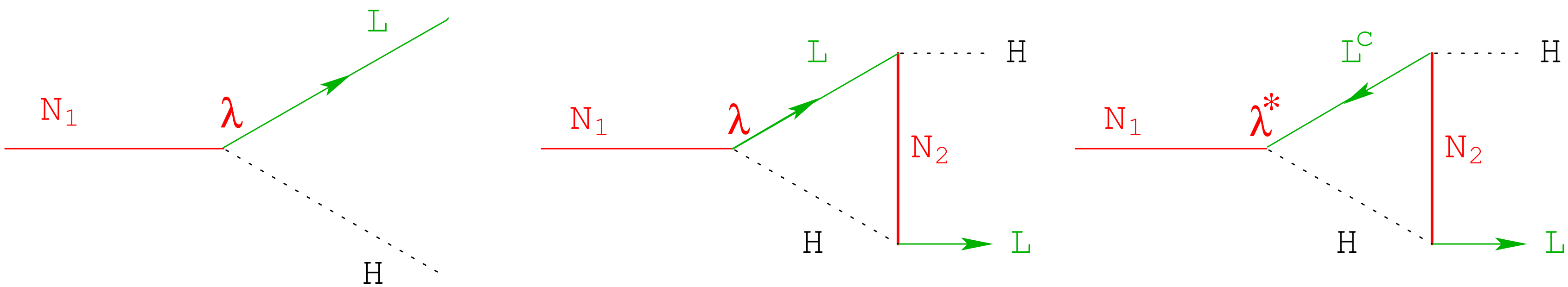
Problems about see-saw Lagrangian

4. Can one obtain the observed mass splittings (see e.g. PDG) by adding only one right-handed neutrino in the three-flavour generalization of the Lagrangian (21)?

5. Generalize the above see-saw Lagrangian (21) for all three SM lepton flavors **and** $\mathcal{N}$ generations of right-handed neutrinos. How many new parameters appears in the see-saw Lagrangian for the case of $\mathcal{N} = 1, 2, 3$?

The simple model

$$\mathcal{L} = \mathcal{L}_{\rm SM} + \bar{N}_1 \partial\!\!\!/ N_1 + \bar{N}_2 \partial\!\!\!/ N_2 + \lambda_1 \bar{N}_1 H L + \lambda_2 \bar{N}_2 H L + \frac{M_1^2}{2} N_1^2 + \frac{M_2^2}{2} N_2^2 \qquad (22)$$

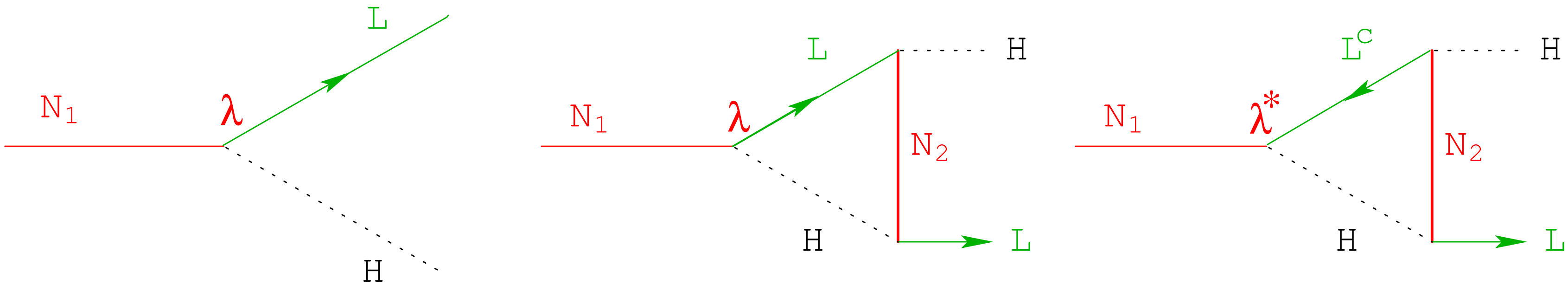


Tree level decay of $N_1 \to L + H$ (the first graph):

$$\Gamma = \frac{|\lambda_1|^2 M_1}{8\pi}$$

– complex phase does not contributes!

CP-violating processes



Sum of the matrix elements of all three graphs gives:

$$\Gamma(N_1 \to LH) \propto |\lambda_1 + A\lambda_1^*\lambda_2^2|^2, \qquad \Gamma(N_1 \to \bar{L}H^*) \propto |\lambda_1^* + A\lambda_1\lambda_2^{2*}|^2$$

where A is some CP-conserving number

$$\frac{\Gamma(N_1 \to LH) - \Gamma(N_1 \to \bar{L}H^*)}{\Gamma(N_1 \to LH) + \Gamma(N_1 \to \bar{L}H^*)} \propto \mathrm{Im}(\lambda_2^2)$$