Test problems

Important information

The purpose of this test is not to give you a grade, and it will not count toward your end result. It is only to estimate the level of your knowledge. The problems will be discussed at the next lecture.

The test consists of 2 parts and the homework. During the test you should try to solve the problems from the first two parts only. You should try to solve the homework before the next lecture.

In all the problems explanations are needed! (if not stated otherwise)

Part 1

Problem 1.1: Let

$$f(\varepsilon) = \frac{1}{\mathrm{Log}(1+\varepsilon)} \tag{1}$$

where Log is a natural logarithm. Calculate Taylor expansion of f up to 3^{rd} order in ε.

Problem 1.2: Assume $\boldsymbol{A} = (1, 2, 3)$ and $\boldsymbol{B} = (2, 3, 4)$. Find:

1. Scalar product $(\boldsymbol{A} \cdot \boldsymbol{B})$;
2. Cross product $[\boldsymbol{A} \times \boldsymbol{B}]$ (we will also call it vector product);
3. Such vector $\boldsymbol{C}$ that $\boldsymbol{A} + \boldsymbol{B} + \boldsymbol{C} = \mathbf{0}$;
4. Double cross product $[\boldsymbol{A} \times [\boldsymbol{B} \times \boldsymbol{C}]]$.

Problem 1.3: Suppose particle of mass m on the spring with rigidity k, that can move only in direction parallel to the spring. Using Newton's laws write equation of motion of particle and solve it.

Problem 1.4: Muon is a particle with decay time $\tau = 2.2 \cdot 10^{-6}$ s (in muon's rest frame). Assume a muon with energy equal to 10 muon masses. What distance will this muon travel before decay?

Part 2

Problem 2.1: Suppose $\boldsymbol{A}(x, y, z) = (\sin(y), e^{x+y}, \cos(xz))$ is a vector field. Calculate:

1. $\mathrm{grad}(\mathrm{div}\boldsymbol{A})$
2. $\mathrm{div}(\mathrm{rot}\boldsymbol{A})$

3. $\mathrm{rot}(\mathrm{rot}\boldsymbol{A})$

Problem 2.2: Find kinetic energy of the particle of mass m in cylindrical and spherical coordinates.

Problem 2.3: Suppose a particle of mass m moving on the spherical surface of radius R in the gravitational field $\boldsymbol{g}$. What are conservation laws held for it?

Problem 2.4: Write down Maxwell equations (all that you remember). Take:

$$\boldsymbol{E} = -\boldsymbol{\nabla}\varphi_0 - \frac{1}{c}\frac{\partial \boldsymbol{A}}{\partial t} \tag{2}$$

$$\boldsymbol{B} = \mathrm{rot}\boldsymbol{A} \tag{3}$$

and rewrite Maxwell equations using 4-vector $A^\mu = (\varphi, \boldsymbol{A})$. Then introduce $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ and rewrite Maxwell equations using it.

Homework

Problem H.1: Block of mass m stands still on the wedge with slope θ from the horizon in gravitational field $\boldsymbol{g}$. Draw the picture of all forces acting on it. Find values of these forces in equilibrium.

Problem H.2: Billiard ball of speed v strikes another billiard ball of the same mass, that stands still. Show, that for elastic non-central collision angle between speed vectors of balls after collision is equal to $90°$.

Problem H.3: Write formula for energy of relativistic particle and make an expansion in v/c up to 3^{rd} order.

Problem H.4: Suppose an integral

$$S = \int_{t_1}^{t_2} L\left(f(t), \dot{f}(t)\right) dt \tag{4}$$

where $L(x, y)$ is some function of 2 arguments and $f(t)$ is an arbitrary function. Result of this integral depends on particular choice of $f(t)$, so we will write $S[f(t)]$. In mathematics this construction is called *functional*. Show, that condition of minimal (or maximal) values of S is:

$$\frac{\partial L}{\partial f} - \frac{d}{dt}\frac{\partial L}{\partial \dot{f}} = 0 \tag{5}$$