Homework 1

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Problem 1.1: Given a system of N particles with Cartesian co-ordinates $\vec{r}_1, \ldots, \vec{r}_N$, introduce generalized co-ordinates $q_1, \ldots, q_n$, $n \leq 3N$. The relations between old and new co-ordinates are:

$$\vec{r}_i = \vec{r}_i(q_1, \ldots, q_n, t) \tag{1}$$

(a) Prove that

$$\frac{\partial \vec{v}_i}{\partial \dot{q}_j} = \frac{\partial \vec{r}_i}{\partial q_j} \tag{2}$$

where $\vec{v}_i = d\vec{r}_i/dt$;

(b) Prove that

$$\frac{\partial \vec{v}_i}{\partial q_k} = \frac{d}{dt}\frac{\partial \vec{r}_i}{\partial q_k} \tag{3}$$

Problem 1.2: **Variation technique** Using variational methods, show that the straight line is the shortest way between two points in space.

Problem 1.3: **Modified random walk** Suppose random walk of the particle on the lattice with step l and time step τ. Evolution of the system is given by (see lecture):

$$P_{N+1}(m) = qP_N(m-1) + (1-2q)P_N(m) + qP_N(m+1) \tag{4}$$

with $q < 1/2$. Using continuous approximation $l, \tau \to 0$, $l^2/\tau \to const$, find diffusion equation. How does diffusion constant D depend on q?