Thermal fluctuations in the microwave conductivity of $Bi_2Sr_2CaCu_2O_8$

M. L. Horbach and W. van Saarloos
Institute-Lorentz for Theoretical Physics, University of Leiden, P.O. Box 9506, 2300 RA Leiden, The Netherlands
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Recently, a peak close to T_c has been observed in the microwave conductivity of single crystals of the high-temperature superconductor $Bi_2Sr_2CaCu_2O_8$. This peak was interpreted as a coherence peak. In this paper we investigate an interpretation in terms of thermal-fluctuation effects. The fluctuation contribution to the conductivity calculated by Aslamasov and Larkin (generalized to finite frequencies by Schmidt) is of the magnitude of the observed effect, but leads to a narrow peak at T_c. In microwave experiments in the gigahertz range, however, which probe a surface layer with a distribution of T_c's, thermal fluctuations lead to a broader peak slightly below the dc critical temperature, as observed. Strong pair breaking tends to shift this peak somewhat further below the dc critical temperature, and also suppresses the importance of other fluctuation contributions (Maki-Thompson), which in principle could lead to a fluctuation peak in the nuclear-spin-relaxation rate. Our results are consistent with the conclusion that there are no true coherence peaks in the conductivity or nuclear-spin relaxation as a result of strong pair breaking.

I. ABSENCE OF COHERENCE PEAKS

A well-known characteristic feature of weak-coupling BCS superconductors is the existence of a so-called coherence peak[1,2] in both the nuclear-spin-relaxation rate $1/T_1T$ and the microwave conductivity $\sigma_1(\omega)/\sigma_{1n}(\omega)$ at frequencies much smaller than $\Delta(0)$, the gap at zero temperature. The peak in these quantities as a function of temperature appears typically at about $0.8T_c$ and has a width of about $0.4T_c$.

According to BCS theory, a coherence peak reflects properties of the quasi-particle spectrum and of the singular nature of the weak-coupling quasiparticle density of states in the superconducting state just above the gap, $N_s(E)=EN_n(E)/\sqrt{E^2-\Delta^2}$, where $N_{s(n)}(E)$ is the quasiparticle density of states in the superconducting (normal) state.

It is an established experimental fact[3] that the high-T_c cuprate superconductors show no peak below T_c for $1/T_1T$. Several explanations for this have been put forward. Strong inelastic scattering[4] (with a bosonic mode which presumably is of electronic origin[5]) leads to a smearing of the density-of-states singularity and filling of the gap and, depending on the strength of the coupling, to the suppression of the coherence peak. An implication of such an explanation is that the coherence peak in the microwave conductivity should be absent as well, when measured at sufficiently low frequencies.

Several groups,[6-8] however, have observed peaks in the conductivity at frequencies up to 60 GHz. These peaks look rather different from the weak-coupling BCS coherence peaks, in the sense that they occur very close to T_c and that they are very narrow, the width typically 1– 3 K (see inset of Fig. 3). Also this could be due to strong coupling, as was suggested by Holczer *et al.*[6] A large value of $2\Delta(0)/kT_c$ and a constant $\Delta(T)$ from zero temperature almost up to T_c results in a narrow coherence peak (Fig. 1). Large pairbreaking, however, as it usually occurs in the case of strong coupling, would destroy this peak. In any case, it seems that strong coupling cannot explain both the absence of a peak in the $1/T_1T$ data and the presence of a narrow peak in the conductivity.

A fit of the narrow conductivity peak observed on granular $YBa_2Cu_3O_7$ films by Kobrin *et al.*[9] within weak-coupling BCS theory was presented in Ref. 10, with as ingredients a temperature-dependent mean free path and effective carrier mass and a temperature-dependent mixture of normal and superconducting regions of the sample close to T_c. A large mean free path near T_c suppresses the coherence peak, while the mixture of normal and superconducting regions leads to the appearance of an additional peak.

A mechanism which has different effects on the nuclear-spin relaxation and the conductivity is provided by the spin-bag approach. In this approach a collective mode exists which renormalizes the interaction which is

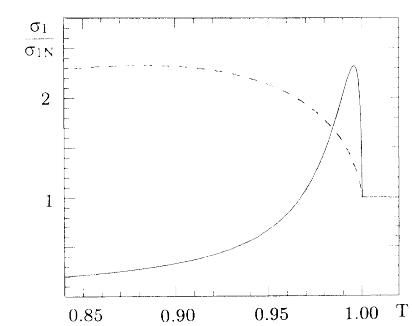

FIG. 1. A narrow coherence peak obtained for $2\Delta(0)/kT_c=9$ and a gap which is temperature independent up to about $0.85T_c$. Both anomalous features are necessary to produce such a narrow peak. The dashed line is a weak-coupling BCS coherence peak. Horizontal axis in units of T_c.

relevant for the nuclear-spin relaxation, but which does not affect the conductivity. Consequently this approach predicts[11] a coherence peak in the conductivity but not in the nuclear-spin relaxation. If the narrow conductivity peak is indeed a coherence peak it might be viewed as experimental support for this picture.

It must be mentioned that at relatively high frequencies (THz) a broad conductivity peak has been observed.[12] An explanation for this peak has been given[12,13] in terms of a competition between an increased quasiparticle lifetime τ (and consequently an increased diffusion constant $D=v_F^2\tau/d$) when the temperature is lowered through T_c and a decrease of the density of states N_s in the gap region. The details of this mechanism appear to depend sensitively on the opening up of the gap as a function of temperature. In particular, pair-breaking effects can lead to gapless superconductivity. The precise way in which a real gap is filled in, related to the pair-breaking rate, then determines whether the decrease of the quasiparticle density of states close to the Fermi energy is lowered sufficiently in order to overrule the increase of the diffusion constant. This delicate mechanism would not lead to a peak in the nuclear relaxation rate, consistent with experiment.

Recently, Marsiglio[14] found that, within the framework of Eliashberg theory, the conductivity coherence peak (not the $1/T_1T$ peak) disappears in the clean limit. In view of Holczer's[6] observation of a peak in the conductivity, he concludes that the clean limit can be ruled out. Also, based on the same argument, he rules out very strong coupling, since then the peak disappears. These statements of course depend heavily on the interpretation of the observed conductivity peaks as coherence peaks.

We shall concentrate our attention on these narrow conductivity peaks. Our aim is to show that the peak in $Bi_2Sr_2CaCu_2O_8$ observed at 60 GHz by Holczer *et al.*, rather than being coherence peaks, might well be due to thermal fluctuations. The picture then is that although a real coherence peak is absent, presumably due to strong-coupling effects, a fluctuation-induced peak may arise. In the experiments on $YBa_2Cu_3O_7$ at lower frequencies,[7,8] however, peaks near T_c are not predominantly due to fluctuation effects. As we will discuss, the large peaks near T_c at low frequencies found in some experiments in these materials are presumably due to an experimental artifact identified by Olsson and Koch.[15] This mechanism may also play a role in $Bi_2Sr_2CaCu_2O_8$, however.

The importance of thermal fluctuations for the high-temperature superconductors in general is suggested by the effective two-dimensionality of the Cu-O layers, the high transition temperature, and the short coherence length. Fluctuations affect the conductivity and the nuclear-spin relaxation rate. Strong pair breaking has a small effect on certain fluctuation contributions (Aslamasov-Larkin diagram[16]) while it suppresses others (Maki-Thompson diagram[17,18]). Since the nuclear-spin relaxation, unlike the conductivity, is not affected by the former type of diagram,[19] we do not expect a large effect of thermal fluctuations on the nuclear-spin relaxation rate.

We shall see that, although fluctuations lead to a peak in the conductivity right at T_c, a distribution of T_c's leads to a peak below the dc critical temperature, in agreement with experiment.[6]

II. FLUCTUATION-INDUCED CONDUCTIVITY PEAK

Above the critical temperature, lowering of the temperature leads to the anticipation of the superconducting state due to thermal fluctuations and thus the dc resistance decreases. The contribution to the static conductivity from fluctuations of the order parameter was calculated by Aslamasov and Larkin[16] and generalized for the frequency-dependent conductivity by Schmidt.[20] We will focus on this contribution here, and will discuss other terms (such as the Maki-Thompson term) later. The result of Schmidt, for the real part σ' of the conductivity of films with thickness $d<\xi(T)$, where $\xi(T)$ is the correlation length of the fluctuations, is given by

$$\sigma'(\omega)=\frac{e^2}{16\hbar d\epsilon}\left[\frac{\pi}{\omega'}-\frac{2}{\omega'}\arctan\left[\frac{1}{\omega'}\right]-\frac{1}{\omega'^2}\ln|1+\omega'^2|\right]\quad(T>T_c)\;,\qquad(1)$$

where $\epsilon=(|T-T_c|)/T_c$ and $\omega'=\hbar\pi\omega/16k_BT_c\epsilon$.

Below T_c the effect of variations of the order parameter around its nonzero mean-field value on the real part of the frequency-dependent conductivity were also studied by Schmidt,[21]

$$\sigma'(\omega)=\frac{e^2}{4\hbar d\epsilon}\left\{\frac{\omega''}{(1+\omega''^2)}\left[\frac{\pi}{2}-\arctan\left[\frac{1}{\omega''}\right]\right]-\frac{1}{2(1+\omega''^2)}\ln\left[\frac{1+\omega''^2}{4}\right]\right\}\quad(T<T_c)\;,\qquad(2)$$

where $\omega''=2\omega'$.

The expressions (1) and (2) have the scaling form $\sigma'(\omega)\propto(1/\omega)F(\omega/\epsilon)$. The function $F(x)$ goes to a nonzero constant for large values of x, while it is proportional to x for small x.

This results (1) and (2) join at T_c, leading to a maximum fluctuation contribution $\sigma'_{2D}(T_c)=(e^2k_BT_c)/(\hbar\omega\hbar d)$ at T_c. Note, however, that due to the logarithmic terms $d\sigma'/dT$ is infinite at T_c.

The width ΔT of the peak at half of its height follows from the criterion $(\hbar\pi\omega)/(8k_B\Delta T)\approx1$. The value of the function $F(x)$ then is approximately half of its limiting value for large x, i.e., its value at T_c. For a frequency of 60 GHz this yields $\Delta T\approx1$ K. Below the GHz regime the width is unmeasurably small.

Although the analysis of Schmidt is based on a Gaussian fluctuation theory, a simple scaling analysis[22,23] shows that the $1/\omega$ behavior at T_c actually holds more generally. In three dimensions, σ' behaves as $1/\sqrt{\omega}$ at T_c in the Gaussian theory, and as $\omega^{-1(1/z)}$ according to the scaling theory. If, as argued by Fisher, Fisher, and Huse,[22] the so-called model A relaxational dynamics with $z\approx2$ applies, this gives a similar exponent.

Equations (1) and (2) have been derived for a homogeneous film of thickness d. In the cuprates the conduction takes place in the Cu-O layers, which are only weakly coupled. This is especially true for $Bi_2Sr_2CaCu_2O_8$. As will be discussed further below, except for a region extremely close to T_c, this material behaves essentially two dimensionally. The distance d should then be taken as the distance between the Cu-O layers.[24]

In Fig. 2 we show the results of the computation of $\sigma_1(\omega)/\sigma_{1n}(\omega)$ with the 2D fluctuation effects taken into account.[25] The normal-state conductivity is taken to be temperature independent, and the fluctuation contribution is added to a behavior without a coherence peak, as could be the result of strong-coupling effects. The latter is indicated by the dashed line. The drop of the mean-field behavior below T_c is rather drastic in Fig. 2. Whether this is really the case in the cuprates depends on the details of the strong-coupling effects. A more smooth behavior is possible.[4] The solid line is the result including the Aslamasov-Larkin-Schmidt fluctuation contribution. The frequency is taken to be the frequency at which the experiment of Ref. 6 (on $Bi_2Sr_2CaCu_2O_8$) was performed, 60 GHz, and the normal-state resistance per square was taken to be 300 Ω, a value reported for $Bi_2Sr_2CaCu_2O_8$ in Ref. 26. Thus, without adjustable parameters for the fluctuation contribution, the conductivity enhancement we find is close to the height of the peak, which is observed experimentally. In fact, the predicted peak value 2.9 is higher than the experimental value of 1.9. Also notice that at frequencies of the order of 60 GHz the fluctuation peak at half of its height is of the order of 1 K, while at lower frequencies the peak becomes very narrow, as mentioned before. As discussed above, at frequencies much smaller than 60 GHz, fluctuation peaks become extremely narrow (in the absence of any broadening effects). At frequencies much higher than 60 GHz, on the other hand, the height of the peak is too small to lead to observable effects.

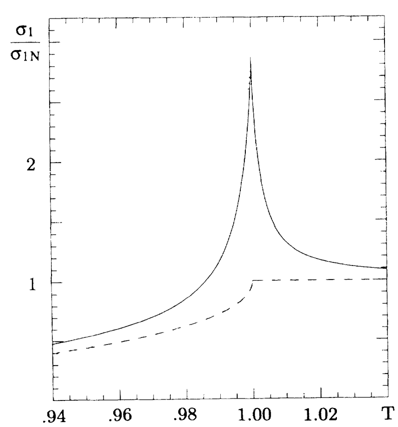

FIG. 2. Fluctuation-induced peak, at a frequency of 60 GHz. The normal-state resistance per square of a Cu-O layer is taken to be 300 Ω. The fluctuation contribution is superimposed on a mean-field behavior with a suppressed coherence peak, indicated by the dashed line, presumably due to the strong-coupling effects. The dotted curve is the result with the pair-breaking parameter $\rho=0.2$.

Of course, the fluctuation-induced conductivity peaks right at T_c, while the experimentally observed peak is slightly below T_c. We will come back to this in Sec. IV, where we will argue that in the presence of a distribution of T_c's the fluctuation peak occurs slightly below the dc critical temperature.

Since the Cu-O planes are weakly coupled, a crossover to three-dimensional behavior is expected close to T_c. Within the Lawrence-Doniach[23] model, in which the coupling is the Josephson type, the crossover occurs when the correlation length in the direction perpendicular to the Cu-O layers, $\xi_z(T)$, becomes comparable to the distance between the Cu-O layers. In the case of a high anisotropy $\xi_z(T)/\xi_{xy}(T)$ is very small, smaller than 0.02 in $Bi_2Sr_2CaCu_2O_8$.[22] The crossover to three dimensions then occurs immeasurably close to T_c, typically at $0.9995T_c$. Right at T_c, where ξ_z diverges, the fluctuation contribution to $\sigma'(\omega)$ for the anisotropic three-dimensional case is the three-dimensional Aslamasov-Larkin result enhanced by the anistropy factor ξ_{xy}/ξ_z. This crossover from the 2D to 3D fluctuation conductivity has been observed experimentally in the dc resistivity vs temperature of $YBa_2Cu_3O_7$ at about 1 K away from T_c,[27] but, as mentioned above, for $Bi_2Sr_2CaCu_2O_8$ this crossover is unimportant. Note, however, that due to the smaller normal-state conductivity of $YBa_2Cu_3O_7$ the effect of the smaller anisotropy, which leads to a smaller peak in σ'_1/σ_{1n} due to a crossover to three-dimensional fluctuations, is partially undone. In particular, at 60 GHz the fluctuation effect might still be measurable in $YBa_2Cu_3O_7$. Indeed, in Ref. 9 a peak of height 1.9 in σ'_1/σ_{1n} for $YBa_2Cu_3O_7$ at 58.9 GHz was reported.

Recently, experiments[7,8] have been performed on $YBa_2Cu_3O_7$ thin films for frequencies between 50 kHz and 500 MHz, which yield very sharp enhancements in σ' slightly below T_c. The observed $1/\omega$ frequency dependence of the peak height does not agree with the three-dimensional Aslamasov-Larkin-Schmidt formula,[20] which yields a $1/\sqrt{\omega}$ behavior. Furthermore, as mentioned before, at frequencies lower than the GHz regime the fluctuation-induced peak is, without broadening due to a distribution of T_c's extremely narrow. These effects seem therefore not due to fluctuations. As discussed further in Sec. IV, these peaks are likely to be an experimental artifact.

III. EFFECTS OF PAIR BREAKING

In line with the fact that pair breaking becomes important for strong coupling, pair-breaking effects play a role in the high-temperature superconductors: it has been estimated[29] that the actual T_c is a factor of 2 lower than what it would have been without pair breaking. Pair breaking can suppress the coherence peak, but it does not affect the Aslamasov-Larkin-Schmidt fluctuation contribution (the Cooper-pair conductivity) above the critical temperature. The Maki-Thompson contribution[17,18] (the contribution of electron-hole pair scattered into another electron-hole pair by exchange of a Cooper-pair propagator), however, is known to be sensitive to large pair breaking. For instance, experiments on aluminum films[19] show a sharpening of the resistive transition with the addition of magnetic impurities or with the application of a parallel magnetic field, both of which are pair breaking effects for BCS superconductors. For small pair breaking the Maki-Thompson contribution is, well away from T_c, typically one order of magnitude larger than the Aslamasov-Larkin contribution. Consistent with our assumption that the mean-field coherence peak is absent due to strong coupling, it is consistent to neglect Maki-Thompson-type fluctuation contributions. In any case, these contributions would enhance the conductivity even more.

Maniv and Alexander,[29] and more recently Kuboki and Fukuyama,[30] have predicted that fluctuations can also enhance the nuclear-spin relaxation rate. However, the enhancement is typically expected to be weak. Moreover, Aslamasov-Larkin-type diagrams do not occur in the local spin susceptibility, which is measured in the nuclear-spin relaxation. Only Maki-Thompson type diagrams determine the fluctuation correction in this case.[28] The contribution of the latter is suppressed in the presence of strong pair breaking. Possibly this is the reason that no fluctuation-induced peak is observed in the NMR experiments.

Below T_c pair breaking does affect the result (2); it naturally enhances the fluctuation effect. This results[21] in a change of the characteristic time scale of the fluctuations, $\tau_{\rm GL}=(\pi\hbar)[16k_B(T-T_c)]$, to $\tau_{\rm GL}/f(\rho)$, where

$$f(\rho)=\frac{\pi^2[1-\rho\psi'(\rho+\frac{1}{2})]}{2\psi'(\rho+\frac{1}{2})} . \quad (3)$$

Here $\psi'(x)$ is the trigamma function[31] and ρ is a pair-breaking strength parameter, which is related to the quasiparticle scattering time τ_0 by $\rho=\hbar/(4\pi\tau_0 k_B T)$. With τ_0 of the order of 10^{-14} (the value determined experimentally in Ref. 32), ρ is close to T_c of the order 0.1.

The effect of pair breaking on the fluctuation-induced conductivity peak is indicated in Fig. 2. Here we have taken a temperature-dependent pair breaking proportional to $(T/T_c)^3$, as was considered in Ref. 33 in an analysis of the suppression of the NMR coherence peak due to pair breaking. The fluctuation contribution is enhanced below T_c.

IV. DISTRIBUTION OF T_c's

In a homogeneous sample with one critical temperature, the fluctuation contribution to σ' peaks at T_c, whereas in the experiment by Holczer *et al.*[6] a peak is observed slightly below T_c. We shall argue that inhomogeneities that lead to a distribution of T_c's in a microwave experiment naturally shift the fluctuation peak to below the critical temperature as obtained in a dc measurement.

The T_c reported in Ref. 6 was obtained from a dc resistivity measurement, which yields the highest temperature for which there exists a percolating superconducting path in the bulk of the sample. In a microwave experiment, on the other hand, a surface layer is probed which is expected to be of a poorer quality than the bulk. Therefore, the dc-transition temperature, which we denote by $T_c^{\rm dc}$, lies on the high-temperature side of the distribution of T_c's in the surface layer. The thickness of the surface layer is given by the penetration depth, which is thicker for lower frequencies.

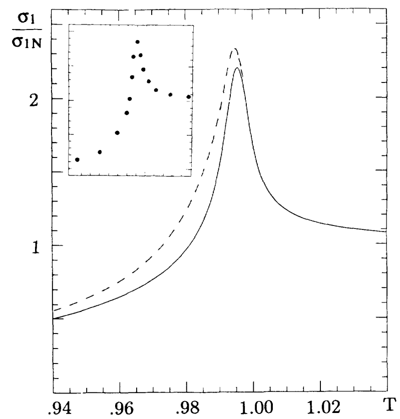

FIG. 3. Fluctuation-induced peak in case of a distribution of T_c's. The horizontal axis is given in units of $T_c^{\rm dc}$. The dashed curve is obtained with the pair-breaking parameter $\rho=0.2$. The inset shows some of the data points of Ref. 6 from $T=74$ to 100 K, with peak height of 1.9 and $T_c^{\rm dc}=91$ K.

In Fig. 3 we show results for the case of a Gaussian distribution of T_c's from $0.99T_c^{\rm dc}$ to $T_c^{\rm dc}$. Except for the width of the distribution of critical temperatures, also the precise form of the mean-field strong-coupling curve influences the width of the peak.

Recently, Olsson and Koch[15] have pointed out that a distribution of critical temperatures can also give rise to a peak in σ' when calculated from the measured complex impedance, which involves both the real part σ' and the imaginary part σ''. Below T_c, σ'' has a contribution of the form $\rho_s/i\omega$ from the superfluid condensate with density ρ_s. Since only the total impedance of the sample is determined, σ' and σ'' from regions below and above T_c get strongly mixed. The width of the resulting apparent peak in σ' appears to be roughly the same as the width of the distribution of T_c's. Unfortunately, this additional complication will make it quite difficult to disentangle a fluctuation peak from such nonintrinsic behavior without independent information on the sample quality, especially since the above effect depends both on the distribution of T_c's and the behavior of ρ_s.

V. CONCLUSIONS

In this paper, we have focused mainly on the experiments by Holczer *et al.*[6] on single crystals of the highly anisotropic material $Bi_2Sr_2CaCu_2O_8$. For this material, and for the frequency they used (60 GHz), we find that the 2D Aslamasov-Larkin fluctuation contribution is of the same order as the peak which is seen experimentally. The location of the peak, slightly below the critical temperature as obtained from a dc-resistivity measurement, we attribute to the fact that in a microwave experiment a surface layer is probed, which has a poorer quality than the bulk.

Low-frequency experiments[7,8] (50 kHz–500 MHz) on thin films of the less anisotropic $YBa_2Cu_3O_7$ also yield peaks just below T_c^{dc}. The frequency dependence of their magnitude is inconsistent with the 3D fluctuation conductivity. Also, the shift of the peak to below T_c^{dc} as we described does not apply for these low-frequency experiments on this films, since the penetration depth exceeds the film thickness.[8] Olsson and Koch have observed that sample inhomogeneities can give an apparent peak in σ' as a function of temperature. This effect possibly plays also a role in Holczer's experiment. Therefore, the precise origin of the enhancement seen in $Bi_2Sr_2CaCu_2O_8$ can only be determined by a more precise analysis of the data (for instance, the frequency dependence) and the sample quality. But in any case, we have shown that the fluctuation enhancement is a large effect in $Bi_2Sr_2CaCu_2O_8$. Nevertheless, whatever the relative importance of the two effects is, it appears justified to conclude that, contrary to the authors'[6] interpretation of their data, these provide no evidence for the existence of a coherence peak.

The Gaussian theory, i.e., the theory of noninteracting Cooper propagators, the modes which signal the instability of the normal state and which drive the phase transition, is valid only outside the critical region around T_c. In the critical regime the Cooper-pair propagator is renormalized due to its self-interaction. As discussed in more detail by Fisher, Fisher, and Huse,[22] critical fluctuations may be observable in $Bi_2Sr_2CaCu_2O_8$. Our analysis shows that in sufficiently high-quality $Bi_2Sr_2CaCu_2O_8$ single crystals, in which the effect discussed by Olsson and Koch disappears, the fluctuation peak is large and measurable. For such samples it may then be possible to see the effects of critical fluctuations in the temperature dependence of σ'.

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